ESTIMATE OF MOUNTAIN SLOPE STABILITY FROM THE MATHEMATICAL MODELING DATA

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Using the data on shear tests of sandy and clayey soils as the base, a mathematical model is proposed for estimating the stability of mountain slopes by solving the inverse problem. The initial information is assigned from the results of acoustic sounding of the region of slip surface origination.

Mountain slope, stability, inverse problem, acoustic sounding

INTRODUCTION

For many regions of the world, the mountain slope stability is the problem of paramount importance. Suffice it to bring to mind the disastrous effects produced by landslides in Italy, China, South Korea, Georgia, and Kirgizia. One of the methods for increasing the safety of submontane territories is monitoring of the most dangerous slope sections whose location can be determined on the basis of geological investigations or by retrospective analysis.

The problems of monitoring include determination of the state and properties of an object (for the subsequent behavior prediction with the help of mathematical modeling), as well as the tendencies to their variation. For the solution of the first problem, part of the initial data can be obtained from experimental investigations of soil and rock samples; for the second problem — by highly precise geodetic measurements of surface point displacements. However, in this case, the difficulties arise, since the relative displacement \( R \sim 10^{-4} \text{ m} \) has an irreversible character, which is established in the laboratory and in-situ tests on shearing the blocks of rocks and soils (sample dimensions are from 0.05 to 2 m) [1, 2]. If such values can be recorded in great base lengths, then the onset of block movement or soil-mass part slipping is characterized by displacements which are less by an order of magnitude. Therefore, it is expedient to pass from the passive methods of slope stability control to the active ones, among which acoustic sounding is widely used.

It is known that the narrow zones, where the irreversible deformations are localized, form in the originally homogeneous material under shear till a certain level of load is reached. In mathematical model, these zones are considered as the slip lines or planes of parts into which a body is separated: the Chernov–Lüders lines in metals; faults and disjunctive dislocations in rocks; gliding planes in soil slopes.

1. EXPERIMENTAL DATA, IDEA, AND HYPOTHESES

In Fig. 1 (solid line), the experimental graph is presented for changing the propagation velocity of the longitudinal waves \( V \) in humid sand depending on the shear stress \( T \) applied in the course of shear tests [3]. Here, two phases are distinguished: preliminary compaction (\( V \) is practically invariable, \( T < T_1 \))
and an increase in \( V \) until \( T \) reaches the limit stress \( T_2 \). In Fig. 2, the solid line shows the dependence of signal amplitude \( A \) (in arbitrary units) on \( T \) to be obtained in the same experiments. In the section \( T_1 < T < T_2 \), the value of \( A \) increases, which can be interpreted as a decrease in the medium viscosity \( \eta \). The results of the analogous tests carried out on clays are demonstrated by dashed lines in Figs. 1, 2. The graphs also have two sections, but in the second one \( T_1 < T < T_2 \) (prior to the onset of uncontrollable slip), \( V \) and \( A \) decrease.

Thus, for the wide range of soils, there is a section in \( V-T \) diagram, where practically unambiguous dependence of the wave propagation velocity on the shear stress exists, which makes it possible to estimate theoretically \( T \) by the value of \( V \) measured \textit{in situ}. The amplitude \( A \) is less suitable for this purpose, since the additional calibration of equipment is required.

So let a zone \( D \) appear during the shearing in the soil mass; its physical properties (\( V, \eta \), and the density \( \rho \)) change within the process of deformation. If the loading occurs in the tangential direction under constrained conditions (considerable normal stresses act), then the medium is compacted; in the opposite case, it is loosened, and the volume increases [4, 5]. For the mountain slopes, the first case generally takes place; the change in density is insignificant in the zone \( D \) as compared with the variation in velocity. Therefore, we assume \( \rho = \rho_0 \).

Our goal is to estimate the parameters of \( D \) from the data of acoustic sounding.

### 2. STATEMENT AND SOLUTION OF DIRECT PROBLEM

To substantiate the approach proposed, let us consider the one-dimensional situation. At the left end of the segment \([0, l]\) the source emitting a sounding signal is located:

\[
f(t) = f_1 e^{-\alpha(t-t_0)} \frac{\sin m \beta t}{\sin m \beta t_0} H(t_0-t),
\]

where \( H \) is the Heaviside function; \( t_0 \) is the duration; \( t_1 \) is the time of increase up to the maximum \((f(t_1) = f_1)\); \( \alpha = m \cot \beta t_1 \); \( \beta = \pi / t_0 \); \( m \) is the integer showing the number of continuous derivatives of \( f \) (according to [6], \( m = 4 \)).

The zone \( D \) lies within the segment \([x_0-a, x_0+a]\) (Fig. 3). In the domain studied, the spatial distribution of medium properties is described by the functions

\[
V(x) = V_0 q(v, x),
\]

\[
\eta(x) = \eta_0 q(s, x),
\]

where

\[
q(p, x) = 1 + (p-1) \left( \frac{(x-x_0)^2}{a^2} - 1 \right)^2 H(a-|x-x_0|),
\]

while \( V_0 \) and \( \eta_0 \) are the values of the corresponding parameters in the initial (undeformed) state.

Let the medium be humid sand, then \( \nu > 1 \) and \( s < 1 \).

At small \( f_1 \) the increments in the strains \( \Delta \varepsilon \) caused by the external action are connected with the increments in the stresses \( \Delta \sigma \) by Hooke’s law

\[
\Delta \sigma(x, t) = E(x) \Delta \varepsilon(x, t),
\]

where \( \Delta \varepsilon = \partial u / \partial x \); \( u \) is the displacement; \( E \) is Young’s modulus.