RAYLEIGH-WAVE PROPAGATION IN A HALF-PLANE COVERED WITH A PRESTRESSED LAYER UNDER COMPLETE AND INCOMPLETE INTERFACIAL CONTACT

M. Ozisik* and S. D. Akbarov**

Keywords: Rayleigh wave, initial stresses, layered half-plane, composite, wave propagation, dispersion

Under a plane-strain state in a half-plane covered with a homogeneous prestressed layer, the Rayleigh-wave propagation along the layer is investigated. The investigations are carried out within the framework of a piecewise homogeneous body model with the use of the three-dimensional linearized theory of elasticity. Complete and incomplete contact conditions between the layer and half-plane are considered. Numerical results relating to the dispersion of the waves are obtained for the case where the layer and half-plane are isotropic and homogeneous.

1. Introduction

The investigations of wave propagation in elastic media with initial stresses are highly significant. A great number of them, such as [1-3] and others, are devoted to the influence of initial stresses on the velocity of various types of waves. Systematic presentations and analyses of the studies carried out before 1986 can be found in [4, 5]. The reviews of later researches are given in [6-10]. A considerable part of the results obtained in this field refer to composite materials with initial stresses consisting of a finite number of layers or considered as media with a periodic structure. Up to now, the Rayleigh waves in homogeneous prestressed composite (layered) materials have not been considered. In the present paper, the propagation of a Rayleigh wave in a homogeneous prestressed half-plane covered with a homogeneous prestressed layer under a complete and incomplete contact between them is studied. The dispersion of this wave and the influence of the mentioned prestress on this dispersion are investigated. It is assumed that the materials of the layer and half-plane are isotropic and homogeneous. It should be noted that the dispersion of Rayleigh waves in a layered half-plane (without initial stresses) were studied in [11, 12] and the results of these investigations are detailed in [13].

2. Formulation of the Problem

We consider a half-plane covered by a layer of thickness \( h \) (Fig. 1). The interface of the layer and half-plane is associated with Lagrangian coordinates \( Ox_1x_2x_3 \), which, in the natural state, coincide with Cartesian coordinates. The quantities related to the half-plane and layer are marked by the superscripts \( (2) \) and \( (1) \), respectively. According to [4, 5], the initial stresses are marked by the superscripts \( (m) \) and \( 0 \).

Let us assume that the initial stresses in the layer and half plane are
\[ \sigma_{11}^{(m),0} = \text{const} \neq 0, \quad \sigma_{ij}^{(m),0} = 0, \quad i \neq j \neq 1. \]  
Here, \( \sigma_{ij}^{(m),0} \) are the stress-tensor components and \( m = 1, 2 \).

All investigations in the present paper are carried out within the framework of the three-dimensional linearized theory of elasticity (TDLTE) in a plane-strain state. We consider the case where the initial stress state is determined by the classical linear theory of elasticity and the stresses caused by wave propagation are significantly smaller than the initial stresses \( \sigma_{ij}^{(m),0} \).

According to [4, 5], the equations of TDLTE for the case considered are
\[
\frac{\partial \sigma_{11}^{(m)}}{\partial x_1} + \frac{\partial \sigma_{12}^{(m)}}{\partial x_2} + \sigma_{11}^{(m),0} \frac{\partial^2 u_1^{(m)}}{\partial t^2} = p^{(m)} \frac{\partial^2 u_1^{(m)}}{\partial t^2},
\]
\[
\frac{\partial \sigma_{12}^{(m)}}{\partial x_1} + \frac{\partial \sigma_{22}^{(m)}}{\partial x_2} + \sigma_{11}^{(m),0} \frac{\partial^2 u_2^{(m)}}{\partial x_1^2} = p^{(m)} \frac{\partial^2 u_2^{(m)}}{\partial x_1^2}.
\]

In Eq. (2), the conventional notation is used.

We will consider two types of contact conditions between the layer and half-plane:
- complete contact
  \[ u_i^{(1)} \bigg|_{x_2=0} = u_i^{(2)} \bigg|_{x_2=0}, \quad \sigma_{i2}^{(1)} \bigg|_{x_2=0} = \sigma_{i2}^{(2)} \bigg|_{x_2=0}, \quad i = 1, 2 \]  
- incomplete contact
  \[ u_2^{(1)} \bigg|_{x_2=0} = u_2^{(2)} \bigg|_{x_2=0}, \quad \sigma_{i2}^{(1)} \bigg|_{x_2=0} = \sigma_{i2}^{(2)} \bigg|_{x_2=0}, \quad i = 1, 2 \]  
  \[ \sigma_{i2}^{(1)} \bigg|_{x_2=h} = 0. \]  
  \[ \sigma_{i2}^{(2)} \bigg|_{x_2=0} = 0. \]

According to Fig. 1, in the free face plane of the cover layer, we have
\[ \sigma_{i2}^{(1)} \bigg|_{x_2=h} = 0. \]

Moreover, we assume that the decay conditions
\[ \sigma_{ij}^{(2)} \xrightarrow{x_2 \to -\infty} 0, \quad u_i^{(2)} \xrightarrow{x_2 \to -\infty} 0 \]
are satisfied and the layer and half-plane are isotropic and homogeneous.

The elasticity relations are