OIL RESERVOIR PRODUCTIVITY WITH HORIZONTAL CIRCULAR CRACK FORMED BY HYDRAULIC FRACTURING

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The approximate approach previously developed within the plane problem for calculating the inflow to the oil well after hydraulic fracturing is generalized for the horizontal crack of circular shape. The formulae are derived for estimating the increase in oil recovery and finding the shape of the crack formed by hydraulic fracturing.

Hydraulic fracturing, filtration, productivity, pressure, opening

1. Mathematical formulation of the problem. Let us state the problem on determination of the well oil recovery after the horizontal hydraulic fracturing (HFR) of the reservoir and formation of a circular crack with the radius \( R \), the filtration coefficient \( R_{id} = -\Delta P_{id} / Q_{id} \) (the ratio of permeability to the fluid viscosity), and the profile of opening \( w(r) \). Place it symmetrically relative to the well axis in the plane of symmetry of oil reservoir \( 2h \) in thickness with the filtration coefficient \( k \). According to [1], we shall analyze the problem on inflow to the well in terms of the variables averaged with respect to the reservoir thickness or crack opening depending on the filtration region under consideration. Let \( P(r) \) and \( Q(r) \) denote the pressure and flow rate in the reservoir; \( P_f(r) \) and \( Q_f(r) \) — in the crack, respectively. Introduce \( V(r) \) — the velocity of fluid crossflow between the crack and the reservoir. It is sufficient to consider the flow in the region situated below the horizontal plane of the problem symmetry. The vertical projection of the filtration velocity is positive in this region. The origin of polar coordinate system \( (r, \varphi) \) is located in the median plane of the reservoir on the well axis. Let \( Q_0 \) designate the inflow to the well from the influence zone of the radius \( L \). To simplify writing, the argument \( r \) will be omitted.

In that part of the oil reservoir where the crack is absent, the flow satisfies the condition of flow rate conservation:

\[
Q(r) \big|_{ r > R } = -Q_0 = \text{const}
\]  
(1.1)

by virtue of the assumed impermeability of the boundaries. In the region of conjugation with the crack, the flow is described by the Darcy linear law, the rate of change in \( Q \) along the radial coordinate is determined by \( V \). Let us average the equations of fluid filtration in the reservoir over the thickness \( 2h \) similarly to [1]:

\[
Q = -\frac{c_D}{\mu} r \frac{dP}{dr}, \quad \frac{dQ}{dr} = \frac{c_D}{\mu} \frac{1}{r} \frac{d}{dr} \left( r \frac{dP}{dr} \right) = 2\pi V,
\]  
(1.2)

where \( c_D = 4\pi R_{id} \) is the coefficient of reservoir conductivity.
When formulating the boundary conditions near the crack, our attention should be drawn to permeability of the zone in the vicinity of the bottom hole. Though the HFR efficiency is governed by three basic factors, the most important one is the formation of channel in the contamination zone of reservoir with high permeability: \( k_1 \gg k \). The other important factors are the volume and the shape of the crack. For the typical extent of hydraulic fracturing by several tens of meters, the first factor leads to a multiple increase in productivity depending on the degree of contamination. After HFR, the rise in oil recovery of an ideal reservoir of constant permeability is usually several times lower. Outside the relatively small disturbance zones mentioned above, the decrease in the reservoir pressure depending on the crack length \( R \) obeys the logarithmic law in the first approximation. Therefore, the productivity proportional to the depression (in oil production, this term signifies the difference between the reservoir and the bottom-hole pressure) also depends on \( \ln R \). This circumstance explains a comparatively slight increase in productivity as the crack formed by hydraulic fracturing grows. The possibility for raising the oil recovery, which does not require the additional resources, consists in optimization of the crack shape with the prescribed amount of proppant due to the selection of optimum injection conditions.

Let us formulate the boundary conditions. In the process of well operation, contamination of the zone adjoining the bottom hole rises. This follows from a comparison between the measured primary and current reservoir productivity by the moment of HFR. We restrict ourselves by the case when the reservoir can be considered as an ideal one immediately after the well drilling. Prior to hydraulic fracturing, the depression becomes stable:

\[
\Delta P_{re} = -R_{re} Q_{re} = -(R_{id} + R_{ad}) Q_{re},
\]

where \( Q_{re} \) is the inflow corresponding to \( \Delta P_{re} \), and \( R_{re} \) is the hydraulic resistance.

We present \( R_{re} \) as the sum of initial hydraulic resistance of the ideal reservoir \( R_{id} = -\Delta P_{id} / Q_{id} \), which is found by \( \Delta P_{id} \) and \( Q_{id} \) at the initial stage of oil recovery and the inflow \( Q_{id} \), and accumulated additional hydraulic resistance. Express \( R_{ad} \) in terms of the measurands:

\[
R_{ad} = \frac{\Delta P_{id}}{Q_{id}} - \frac{\Delta P_{re}}{Q_{re}}.
\]

The coefficient \( R_{ad} \) also relates the additional losses of \( \Delta P \) in the colmatage zone to the current flow rate \( Q \):

\[
Q = -R_{ad}^{-1} \Delta P = -\alpha \mu^{-1} \Delta P, \quad R_{ad} = \alpha^{-1} \mu.
\]

(1.3)

In this case, the hydraulic resistance is considered proportional to \( \mu \) with the coefficient \( 1/\alpha \).

In the vicinity of the well, the radial distribution of \( R_{ad} \) is usually unknown, but thickness of the reservoir with increased resistance is in most cases assumed small as compared with \( R \). To analyze the influence exerted by \( R \) on filtration, in mathematical formulation of the problem, we replace the contaminated layer by negligibly fine screen located at the well boundary \( r = r_0 \). The pressure difference occurs on the both sides of the screen:

\[
P \bigg|_r = P_0 = \Delta P.
\]

Thus, the first boundary condition is formulated. Here and further, \( \bigg|_r \) is the limit at \( r > r_0 + 0 \); \( P_0 \) is the bottom-hole pressure. Another boundary condition follows from (1.2) and (1.3) at \( r = r_0 \). We write it by introducing the value \( \beta = \alpha / c_D \) in the form:

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