DEVELOPMENT OF THE RADIAL CRACK ZONE UNDER THE ACTION OF CYLINDRICAL CHARGE EXPLOSION

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The development of radial cracks under the action of pressure in the cylindrical cavity is investigated. The dependences of the crack length on the rock strength, charge radius, and pressure magnitude are obtained.

Failure, explosion, crack, surface energy

This paper presents the solution of the dynamic problem on propagation of radial cracks from the surface of cylindrical cavity after its walls are affected by pressure which later is constant or disappears in time. It is required to find the dependence of the crack length on the medium properties and the parameters of explosion loading: the form and duration of impulse.

In [1, 2], this problem was studied and formulated with regard for the presence of crushing zone between the cavity and radial cracks. The stress distribution was considered quasi-static, i.e., the time-dependence was determined by the factor proportional to pressure at the crushing zone boundary. In the field of stresses, the development of cracks was also calculated in quasi-static approximation. In the article proposed, the crushing zone is not taken into account, i.e., it is assumed that the radial cracks arise immediately on the charge surface. The stresses are estimated by the dynamic theory of elasticity, the conditions of crack propagation depend on the surface energy of failure. Thus, the influence of the dynamics on the radial crack zone behavior is investigated.

Let the plane strain state take place in elastic brittle medium beyond the circle \( r = R_1 \) (\( r \) is the radial coordinate, \( R_1 \) is the radius of charge). Initially, the medium contains the symmetrical system of radial cracks \( l \ll R_1 \) in length which start on the circle contour. Each two neighboring cracks form the angle \( 2\alpha \ll 1 \) (this limitation substantiates the asymptotic theory of crack dynamics [3]). It is assumed that explosion products do not penetrate into the cracks but exert pressure on the cavity walls and generate stress field within the rock; the stresses cause the crack zone growth. As a result, the crack edges are stress-free. Since all cracks are under the same conditions, it will suffice to study the propagation of one of them. Small deviations from the original symmetry will possibly lead to unstable solution.

Consider successively two problems: I) the axisymmetric problem without cracks with the known pressure at the boundary \( r = R_1 \); II) the additional problem on growth in the system of cracks whose edges are loaded by the circumferential stress from the first problem but with opposite sign, and the boundary is stress-free. Since the solution of the problem on crack propagation is extremely complicated in exact formulation [4, 5], the second problem was solved approximately [3]. The radial displacements were assumed equal to zero, and the tangent displacements — to the linear functions of the polar angle \( \varphi \) with the proportionality coefficient \( r \varepsilon(r, t) \) which depends on the radius and the time \( t \). Such approximation is satisfactory for small \( \alpha \). At larger \( \alpha \) it is required to represent the displacement in series relative to the degree of \( \varphi \) with more terms. When the kinetic and potential, including the surface, energies were calculated, the relations describing the crack development were obtained by Hamilton variational principle.
We introduce the dimensionless variables with the following measurement units chosen: the cavity radius $R_1$, the shear modulus $\mu$, and the longitudinal wave velocity $c_1$. Then $c_2 = \sqrt{(1-2\nu)/(2-2\nu)}$ is the transverse wave velocity, $\nu$ is Poisson’s ratio, time is related to $R_1/c_1$, and strains — to $p_0/2\mu$ ($p_0$ is the magnitude of pressure in the cylindrical cavity). The cracks are characterized by the following values: $R$ is the coordinate of tips; $l << 1$ and $L = R - 1$ are the initial and current lengths, respectively.

The axisymmetric problem without cracks reduces to solving the one-dimensional wave equation relative to the radial displacement $u$:

$$\frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} - \frac{u}{r^2} - \frac{\partial^2 u}{\partial t^2} = 0$$

with boundary

$$\frac{\nu}{1-2\nu} \frac{\partial u}{\partial r} + \frac{1-\nu}{1-2\nu} \frac{u}{r} = \frac{p(t)}{p_0}, \quad r = 1,$$

$$u = 0, \quad r = \infty$$

and initial

$$u = 0, \quad \frac{\partial u}{\partial t} = 0, \quad t = 0$$

conditions [6]. Here, $p(t)$ is the function describing the change in the normal stress in the cavity.

The circumferential stresses are calculated from the formula:

$$\sigma_{\varphi\varphi} = \frac{\nu}{1-2\nu} \frac{\partial u}{\partial r} + \frac{1-\nu}{1-2\nu} \frac{u}{r}.$$

Particular calculations were carried out for three loading variants:

a) $p(t) = -p_0 H(t)$, where $H$ is the Heaviside function;

b) the step-function $p(t) = -p_0 H(1-t)$;

c) the sine function $p(t) = -p_0 H(1-t)\sin{\pi t}$.

Here, the negative sign indicates that the stress applied to the cavity walls is compressive.

Set (1)-(3) was solved by the finite-difference method [7]. In problem II, the approach proposed in [3] is used. The unknown value is the average circumferential strain $\varepsilon(r,t)$ of the region limited in the tangential and radial directions by the neighboring cracks and inequality $1 \leq r \leq R(t)$, respectively; it is found from the equation:

$$\frac{\partial^2 \varepsilon}{\partial r^2} + \frac{3}{r} \frac{\partial \varepsilon}{\partial r} - \frac{6(1-\nu)}{(1-2\nu)\alpha^2} \frac{\varepsilon}{r^2} - \frac{1}{c_2^2} \frac{\partial^2 u}{\partial t^2} = \frac{6}{\alpha^2 r^2} \sigma_{\varphi\varphi}, \quad 1 \leq r < R, \quad t > 0$$

and the additional conditions:

$$\frac{\partial \varepsilon}{\partial r} = 0, \quad r = 1;$$

$$\varepsilon = 0, \quad \left( \frac{\partial \varepsilon}{\partial r} \right)^2 \left( 1 - \frac{\dot{R}^2(t)}{c_2^2} \right) = \frac{6\pi}{a^3(1+\nu)} \left( \frac{G}{R} \right)^3, \quad r = R(t);$$

$$\varepsilon = 0, \quad \frac{\partial \varepsilon}{\partial t} = 0, \quad t = 0,$$

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