THE RELATION BETWEEN GAUSSIAN BEAMS AND MASLOV ASYMPTOTIC THEORY

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Résumé: Приложение асимптотической теории Маслова к общему трехмерному смешанному подпространству шестимерного фазового пространства предложено для получения интегральных суперпозиций Гауссовских пакетов и пучков. Суперпозиция плоских волн и лучевой метод являются специальными предельными случаями предложенного подхода. Те же самые высокочастотные асимптотические формулы были ранее получены методом Гауссовых пучков [8].

Summary: The application of Maslov asymptotic theory in a general 3-D mixed subspace of 6-D complex phase space is proposed to obtain the integral superpositions of Gaussian packets and beams. The ray method and the superposition of plane waves (Maslov method of Chapman and Drummond [7]) are special limiting cases of the above mentioned approach. The same high-frequency asymptotic expansion formulae for seismic body waves were derived previously in [8] using the Gaussian beam method.

1. INTRODUCTION

Ray methods and their various modifications and generalizations have proved to be a very effective tool for computing seismic wave fields in laterally inhomogeneous 3-D structures, the characteristic dimensions of which exceed considerably the prevailing wavelength of the source time function. There are two generalizations of ray methods which enjoy increasing interest. The generalizations are the Gaussian beam method [1] and Maslov asymptotic method [7]. The mentioned approaches are coherent and may yield equivalent formulae for computing seismic wave fields. It is only necessary to generalize Maslov method of Chapman and Drummond [7] to complex phase space. The generalization may be very useful for computations. The application of Maslov method in a 3-D subspace of 6-D complex phase space is concisely proposed in this paper.

The same quantities are used in the expressions in this paper as in the expressions for the paraxial ray approximation [3, 5, 6], because dynamic ray tracing [12, 13, 5, 6] has proved to be a very effective tool for evaluating the quantities used. Some quantities used broadly throughout the paper are introduced in Sec. 2. The reader should pay attention to the vector and matrix notations used. The notations are introduced at the beginning of Sec. 2.

The initial conditions for a time-harmonic wave field are assumed to be specified on a curved surface (screen) in terms of a complex-valued amplitude and a phase. A high-frequency asymptotic expansion of the wave field into Gaussian beams was found in [8]. The asymptotic expansion has the form of a two-parametric integral superposition of Gaussian beams and corresponds to the relevant ray approximation in all regions, where the ray solution is sufficiently regular (smooth). The resulting formulae for the expansion expressed in Cartesian coordinates are presented in Sec. 3 of this paper. The formulae involve the ray method and the Maslov method.

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of Chapman and Drummond [7] as special limiting cases. The first presented formula is a general one, the second formula is a special case of the former one for a plane surface. The formulae are redervied in this paper using Maslov asymptotic theory.

The later formula for a plane surface is derived by applying Maslov asymptotic theory in a 3-D subspace of 6-D complex phase space in Sec. 4. The wave field is first found in the form of a three-parametric integral superposition (a method of the third order [7]). The desired two-parametric superposition is then obtained as a special limiting case of the three-parametric superposition, adopting the used eigenfunctions (Gaussian packets) to be infinitely narrow in the direction of one coordinate and integrating with respect to this coordinate. The formula derived for the case of a plane surface is the same as the formula obtained using Gaussian beams. The formula is then generalized for the case of a curved surface using the paraxial ray approximation.

Some simple numerical examples are shown in [4].

2. THE SPECIFICATION OF SOME USED QUANTITIES

The capital-letter indices will take the values 1 and 2, lower-case indices will take the values 1, 2, 3. The indices will have the form of right-hand suffices. For instance \( f(x) = f(x_1, x_2, x_3), f(x_A) = f(x_1, x_2) \) and, for any function \( f(x_i), f_{x_i=0} = f(0, 0, 0), f_{x_0=0} = f(0, 0, x_3) \) may be used. Pairs of identical indices will indicate summing. This means that we shall use the Einstein summation convention for the suffices instead of writing the summation symbol.

\( 2 \times 2 \) matrices with components \( A_{AB} \) will be parallelly denoted by the symbols \( A \) or \( A_{BA} \). \( 3 \times 3 \) matrices will always be described by means of their components. The symbol \( A_{AB}^{-1} \) will indicate the components of the matrix inverse to \( A_{AB} \), i.e.

\[
A_{AB}^{-1} A_{BC} = \delta_{AC}.
\]

where \( \delta_{AC} \) is the Kronecker symbol. In other words, \( \delta_{AB} \) denotes the components of the unit matrix. \( A_{AB}^T = A_{BA} \) denotes the components of the matrix transposed to \( A_{AB} \), \( A^+ \) the matrix Hermite-adjoint to \( A \) and \( A^* = A^+T \) the matrix complex-conjugate to \( A \).

We shall use three important coordinate systems throughout this paper:

1. Cartesian coordinates \( x_i \).
2. Ray coordinates \( \gamma_i = (\gamma_1, \gamma_2, \gamma_3 = \sigma) \), where \( \gamma_A = (\gamma_1, \gamma_2) \) are the parameters of the ray (e.g. the take-off angles at a point source specifying the initial direction of rays or the coordinates along a screen) and \( \sigma \) is the coordinate along the ray connected with the travel time \( \tau \) or with the arclength \( s \) by the relations

\[
\sigma = \sigma_0 + \int_{\tau_0}^{\tau} v^2 \, d\tau = \sigma_0 + \int_{s_0}^{s} v \, ds.
\]

Here \( v \) is the velocity of propagation of the corresponding wave.

3. Orthogonal ray-centred coordinate system along a chosen central ray. This coordinate system and its computation is described in [12, 13] and also in [3].