ELEMENTARY SEISMOGRAMS OF SEISMIC BODY WAVES IN DISSIPATIVE MEDIA

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Summary: Approximate expressions for elementary seismograms of seismic body waves propagating in media with small causal absorption are derived. Special attention is devoted to modulated signals with a smooth envelope, for which especially simple formulae were obtained. The derived expressions give a good picture of all important effects of causal absorption, viz., the frequency dependent exponential decrease of amplitudes, the velocity dispersion related to absorption, and the decrease of the prevailing frequency.

Let us first consider the propagation of a scalar plane wave $U(z, t)$ in a dissipative medium in the direction of the $z$-axis. Assume that $U(0, t) = x(t)$, where $x(t)$ is known. For $t > 0$, the wave is described by the formula

$$
U(z, t) = 2 \text{Re} \int_0^\infty X(f) \exp \left[ -\alpha(f) z + i \Phi(f) z + i 2\pi f(t - \tau_0) \right] df,
$$

where $X(f)$ denotes the complex spectrum of $x(t)$, $\alpha(f)$ is the absorption coefficient, $\tau_0$ and $\Phi(f)$ are given by formulae

$$
\tau_0 = \frac{z}{v(f_0)}, \quad \Phi(f) = 2\pi f \left[ v^{-1}(f_0) - v^{-1}(f) \right],
$$

$v(f)$ being the frequency dependent velocity and $f_0$ an arbitrary reference frequency.

To evaluate (1) we must first specify the model of absorption. In case of causal absorption, the functions $\alpha(f)$ and $\Phi(f)$ cannot be chosen arbitrarily; they are related by the causality conditions. In this paper, we shall use the Futterman model of absorption [6], which has been used successfully in many seismological applications. It would, however, be possible to use any other model, such as the model of Lomnitz [8], Kjartansson's model [7], non-causal models, etc.

For Futterman's model, the dispersion relations yield

$$
\alpha(f) = \frac{\pi f}{2v} \ln \left( \frac{f}{f_0} \right), \quad \Phi(f) = \frac{\pi f}{2v} \ln \left( \frac{f}{f_0} \right),
$$

where $v$ and $Q_0$ are parameters of the model. They can be determined from the known values of the velocity $v(f)$ and the $Q(f)$ factor at any specified frequency $f_0$ (or from the known velocity $v(f)$ and absorption coefficient $\alpha(f)$ at $f_0$). Note that the product of the velocity $v(f)$ and of the $Q(f)$ factor is independent of frequency in Futterman's model, and it equals $cQ_0$ in Eqs (3). For details refer to [6].


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Inserting (3) into (1) yields

\[ U(z, t) = 2 \Re \int_0^\infty X(f) \exp \left[ -\pi ft^* + 2ift^* \ln \left( f/f_0 \right) + i 2\pi f(t - \tau_0) \right] df , \]

where \( t^* \) is a global absorption factor, broadly used in present seismology (see [3, 9, 10]). It is given by formula

\[ t^* = z/(cQ_0) . \]

The factor \( t^* \) is the higher, the longer the travel path \( z \) and the higher the absorption effects.

The evaluation of (4) is, in principle, simple, but rather time consuming. It is necessary to evaluate the integrand for a system of frequencies and then evaluate the Fourier integral (4) numerically. This procedure will make the computation of ray synthetic seismograms for dissipative media (in which the integrals (4) must be evaluated many times) rather slow. It is, therefore, useful to rewrite the integral (4) in the convoluntary form or to look for some approximations directly in the time domain, avoiding the Fourier transform.

We shall first consider various convoluntary forms of (4), both exact and approximate. The first version, fully equivalent to (4), is as follows

\[ U(z, t) = x(t) * A(t - \tau_0, t^* ) , \]

where

\[ A(z, t^*) = 2 \Re \int_0^\infty \exp \left[ -\pi ft^* + i 2ft^* \ln \left( f/f_0 \right) + i 2\pi f(t - \tau_0) \right] df . \]

As \( f_0 \) can be chosen arbitrarily, we do not consider it as a parameter. We must, however, keep in mind that the value of \( \tau_0 \) corresponds to the velocity \( v(f_0) \). We can, e.g., take \( f_0 = 1 \) without any loss of generality.

The function \( A(z, t^* ) \) represents a one-parameter family of curves. It is simple to show that this one-parameter family of curves can be reduced just to a single curve. Taking \( ft^* \) as a new integration variable in (7) we get

\[ A(t - \tau_0, t^* ) = B(\Theta)/t^* , \]

where

\[ B(\Theta) = 2 \Re \int_0^\infty \exp \left( -\pi f + i 2f \ln f + i 2\pi f(\Theta) \right) df , \]

\[ \Theta = [t - \tau_0 - (t^*/\pi) \ln (f_0 t^*)]/t^* . \]

As will be shown later the expression for \( \Theta \) can be also rewritten as follows \( \Theta = [t - x/v(f_0)]/t^* \), where \( f_s = 1/t^* \). See also [4].