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On Alternative Geometries, Arithmetics, and Logics; a Tribute to Łukasiewicz

Abstract. The paper discusses the similarity between geometry, arithmetic, and logic, specifically with respect to the question of whether applied theories of each may be revised. It argues that they can - even when the revised logic is a paraconsistent one, or the revised arithmetic is an inconsistent one. Indeed, in the case of logic, it argues that logic is not only revisable, but, during its history, it has been revised. The paper also discusses Quine’s well known argument against the possibility of “logical deviancy”.

Keywords: Łukasiewicz, revisability, inconsistent arithmetics, Traditional logic, paraconsistency, Quine.

1. Introduction: the Place of Łukasiewicz in the History of Logic

The last hundred years have produced a number of great philosophical logicians; perhaps more than any other period of the same duration. Russell, Carnap, Quine, Prior, Kripke, are just some of the names that come immediately to mind. The Polish logician Jan Łukasiewicz is also one of this number. And when the history of the period is written, he will surely have a distinctive place in it.

Łukasiewicz contributed to philosophy and logic in many ways, with work on the propositional calculus, with an analysis of syllogistic, and, very importantly, with his scholarship on the history of ancient and medieval logic.

1 This paper is based on an invited address given to the conference Łukasiewicz in Dublin, held at University College Dublin, July 1996, to celebrate the work of Jan Łukasiewicz. For some years it was slated to appear in the proceedings of that conference. It would seem that the production of those proceedings has lapsed, so I thought that it should now find another outlet. I considered rewriting it in a form that did not bespeak its origin, but decided, in the end, to leave it in its original form, as my own tribute to Łukasiewicz. Just before the conference was held I learned of the sad and sudden death of another great logician, and my close friend, Richard Sylvan. This written version, as was the spoken version, is dedicated to his memory.

2 In absolute terms anyway; relative to the size of the human population, 1250-1350 was more prolific.

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However, it is his role in the foundations of non-classical logic for which he will, I think, be primarily remembered.³

I think it no exaggeration to say that Łukasiewicz is the foundational figure in modern non-classical logic. Volume I of *Principia Mathematica* was hardly off the press when Łukasiewicz was busy investigating many different ideas. The most fundamental of these was many-valued logic. The idea that there might be systems of modern rigour where sentences may be neither true nor false seems to be entirely his.⁴ One of Łukasiewicz’ major motivations for this enterprise was a concern with modality. Modern modal logic owes more to C. I. Lewis than to Łukasiewicz. None the less, Łukasiewicz’ concerns with modality (as opposed to the conditional) would seem to predate Lewis’.

Łukasiewicz also generalised his finitely many-valued logics to infinitely many values, producing what would later become known as fuzzy logic, beloved of a number of modern AI practitioners. Łukasiewicz did not, himself, found paraconsistent logic, but the possibility of this in clear in his critique of Aristotle on contradiction, and it was one of his students, Jaśkowski, who first articulated a paraconsistent logic in detail.

2. Logics, Geometries and Arithmetics

In producing these alternative logics, Łukasiewicz had the model of non-Euclidean geometries very much in mind. For example, in an essay on many-valued logic.⁵

It would perhaps not be right to call the many-valued systems of propositional logic established by me ‘non-Aristotelian’ logic, as Aristotle himself was the first to have thought that the law of bivalence could not be true for certain propositions. Our new-found logic might rather be termed ‘non-Chrysippean’, since Chrysippus appears to have been the first logician to consciously set up and stubbornly defend the theorem that every proposition is either true or false. The Chrysippean theorem has to the present day formed the most basic foundation of our entire logic.

It is not easy to foresee what influence the discovery of non-Chrysippean systems of logic will exercise on philosophical speculation. However, it seems to me that the philosophical signif-

³ For an account of Łukasiewicz’ work, see Sobociński (1956), Borkowski and Skupecki (1958), Kotarbiński (1958), and Kotarbiński’s introduction to McCall (1967).

⁴ Some of Brouwer’s work on intuitionism predates Łukasiewicz’ work, but intuitionist logic itself was created by Heyting, somewhat later.

⁵ Łukasiewicz (1930), quotation taken from p. 63 of the English translation.