A historical review of achievements in creep theory is given. Primary attention is focused on the phenomenological approach. Different constitutive equations are discussed for primary and secondary creep as well as for creep with damage. New creep problems are examined.

Keywords: creep theory, damage, constitutive equations, analysis of achievements, review

1. Introduction. The 19th century witnessed several accidents that could not be explained then, although strength theory had already been developed in part by that time. In those accidents, loads were not critical. The cause of the accidents was the inelastic behavior of the material at certain temperatures (higher than 0.3 melting points). It was experimentally established that in this case deformation increased whereas the load remained constant. This phenomenon is called creep, meaning that deformations increase in time at a constant load. It was also established that if deformation is kept constant, then stresses decrease. This phenomenon is called relaxation, i.e., decrease in stresses with time. Moreover, there are also other experimental facts (not discussed here) that may be attributed to the time-dependent effects [9, 15, 60, 93, 134]. Such effects are described in papers devoted to the theory of viscoelasticity [66, 73, 112].

Machine parts, vessels, pipes, and other structural elements are made of materials that can be simulated as deformable rigid bodies. It is important for the numerical analysis of complex structures to know how those bodies behave and to be able to describe their behavior mathematically. In this case, it becomes possible to meet strict requirements for safe operation of machines and other units. The difficulty of analysis is due to different mechanical behavior of any material, which drastically complicates its modeling. Even if certain materials have equal chemical compositions, their stress–strain curves plotted in elementary tests (e.g., tension tests) do not coincide because of their dependence on loading level, temperature, and ambient conditions. If materials are dissimilar, then the curves are yet more different. This is because of the microstructure of the materials—the microstructures of metals, ceramics, and polymers were found not to be similar.

If factors affecting the mechanical behavior of materials are restricted to loading and temperature, then the majority of materials will behave elastically under small loads and at room temperature. If the temperature remains constant and the load strongly increases, then the material exhibits instantaneous, yet inelastic response. When the load is moderate and the temperature increases, the response of the material lags behind, i.e., behavior is again inelastic and time plays a definite role. The material is said to possess plastic properties in the former case and to creep in the latter case. That the material behaves inelastically in both cases may be demonstrated through unloading—deformation will not disappear completely under full unloading. This classification complies with that adopted in many manuals of mechanics such as [10, 16, 43, 51, 55, 98, 135].

There are also other points of view on what types of inelastic behavior should be considered [75]. However, examples of classification in manuals of mechanics differ from each other not very strongly, in contrast to manuals and monographs on materials science [46, 68, 69, 109, 128].

Creep theory appeared in the last century. It developed at the end of the 19th and at the beginning of the 20th centuries as an independent division of engineering mechanics, generalizing information on accidents and experimental observations. Real practical problems required data on the behavior of materials at elevated temperatures, yet under moderate loads. It was...
necessary to know both the stress–strain state and durability of structures. The first fundamental study on creep [44] was published in 1910, meeting practical needs. The power engineering, boiler, and other industries put forward problems that could be resolved through scientific studies into the behavior of structures at elevated temperature, since the theories of engineering mechanics available then were not capable of solving these problems successfully. Note that even then it was clear that the classification into rigid bodies and liquids is sometimes absolutely conditional. A material that creeps behaves as a liquid. Also note that beginning from the first publication on creep, the scientific literature split into two groups. The first group includes studies based on materials science (see, e.g., [109]). As a rule, they were devoted to deformation mechanisms, correlation of microstructure with properties, etc. It should be noted that such an approach models the behavior of materials by one-dimensional equations containing many parameters, which are difficult to determine. The second group includes studies dealing with specific engineering problems within the framework of solid mechanics (see, e.g., [113]). It is here where we may find information on the full analysis of the mechanical behavior of real objects, i.e., not only experimental observations but also mathematical models describing the mechanical behavior of materials and methods for analysis of corresponding applied problems, i.e., real structures. As a rule, the stress–strain state is three-dimensional in this case and the constitutive equations used in models are purely phenomenological. Note that computers strengthen greatly the capabilities of solving specific problems.

Papers and technical reports published within several years after the first publication were concerned with engineering problems. The next fundamental step toward the unified theory of creep was made only in 1929: the Norton law was introduced instead of Hooke’s law, which underlies the theory of elasticity. The Norton law was such that the traditional approaches to the solution of applied problems were inapplicable. Among the distinguishing features of this law are (i) a nonlinear form even in the elementary case (a power law) and (ii) differential relationship (includes time derivatives of kinematic variables) between the kinematic and dynamic variables. Moreover, the experimental identification of material characteristics is more difficult here. On the other hand, as indicated in [54], the Norton law could be associated with a certain mechanism in a material—so-called diffusion creep. Note that already in 1933 Odqvist generalized the uniaxial Norton law to the three-dimensional case. He used the theory of invariants and tensor calculus. This law is still widely used in practical problems. The majority of applications of creep theory dated from before the Second World War were concerned with metals and their alloys. Wide experience was gained in determining material characteristics such as the scaling factor and creep exponent. Unfortunately, this law has restrictions [23]. The main shortcoming is that the creep characteristics of a material are valid only within a narrow range of loads and temperatures. For information on a more general theory, the reader is referred to, e.g., [72].

After the Second World War, the application domain of creep theory considerably extended owing to the wide use of polymeric materials (plastics). However, the classical creep theory was incapable of simulating the behavior of plastics. The reason for this was their microstructure—polymers consist of polymer chains of different configurations, in contrast to metals and alloys whose microstructure is polycrystalline.

Thus, it was necessary to lay a new theoretical foundation—the theory of viscoelasticity (Rabotnov’s hereditary theory was developed particularly well) and the method of rheological models (proposed by M. Reiner). Basic information about both theories may be found in [12, 15, 17, 66, 70, 73, 94, 112, 119]. The method of rheological models and the hereditary theory of viscoelasticity appeared simultaneously with a modified creep theory—the tensor representation of classical creep theory.

At the late 50s of the last century, classical equations describing secondary (steady-state) creep on the basis of the Norton law were derived. Unlike other equations proposed at that time, the classical equations can be easily extended to the case of primary (transient) creep. What remained to find was an adequate elementary description of tertiary (accelerated) creep. In 1958, L. Kachanov published his first study on creep with regard for damage [6]. Kachanov’s approach was elementary—he modified the secondary creep equations by recalculating stresses and adding an evolutionary equation for damage (in his original paper, Kachanov introduced continuity rather than damage). Thus, a new division of continuum mechanics appeared—continuum damage mechanics (CDM) [87]. This division has been developed intensively during the last forty years. The theory has not been completed as yet [99]; however, many practical problems have been solved and a vast body of experimental information has been accumulated. In most cases, proposed models were phenomenological, i.e., they did not accounted for specific information about microstructural changes.

What mechanism of deformation occurs in materials is known to depend on the levels of loading and temperatures. Therefore, mathematical and mechanical models describing the behavior of materials must be different. For example, the authors of [69] discuss reduced load versus reduced temperature diagrams for numerous metals and alloys. Here the load is referred to the shear modulus and temperature to the melting point. These diagrams tell us in what domain one model or another is valid.