NONPERTURBATIVE SPONTANEOUS SYMMETRY BREAKING

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A nonperturbative approach for spontaneous symmetry breaking is proposed. It is based on some conjectural properties of interacting field operators. As the consequences an additional terms like to $m^2 A^2$ appears in the initial Lagrangian.

Key words: nonassociativity, nonperturbative quantization, symmetry breaking.

1. INTRODUCTION

One of the most astonishing results of quantum field theory is spontaneous symmetry breaking. One can say that it is the situation when something arises from the quantization. This means that on the quantum level one has something that was not present on the classical level. Coleman and Weinberg [1] write: "... higher-order effects may qualitatively change the character of a physical theory ... ". The main goal of the Coleman-Weinberg mechanism is to derive a Higgs potential from more fundamental principles, with as few arbitrary parameters as possible. In this mechanism the Higgs potential is induced by radiative corrections, rather than being inserted by hand. In this approach one can sum over higher-loop graphs to induce an effective potential, which may then produce spontaneous symmetry breaking. Of course, it can
be only in the theories with interactions (where we have these higher-loop graphs): that is, a nonlinearity (in Lagrangian) can lead to the interesting consequences for quantized theory. It is not very surprising because on the classical level we have the same: very simple behavior of a classical linear theory can be changed on the very complicated and surprising behavior of a classical nonlinear theory. For example, in nonlinear theories we have monopoles, instantons, black holes, strange attractors and so on. On the quantum level we can expect once more amazing stuffs if we add some nonlinear terms in Lagrangian. Probably one of such manifestations of a nonlinearity is confinement in the QCD.

The problem here is that we do not have detailed techniques for the nonperturbative calculations. Even on the perturbative level we do not sure that the result after the sum over all Feynman diagrams will the same as after the sum over a finite number of Feynman diagrams. Nevertheless, according to perturbative calculations we know that they change an initial Lagrangian so that an extra potential term arises in the Lagrangian (Coleman-Weinberg mechanism).

In this paper we work out a nonperturbative method which can be applied for strongly interacting fields and to show that extra terms (or nonperturbative spontaneous symmetry breaking) appear in an initial Lagrangian if the product of field operators have some properties concerning to the rearrangement of the brackets in a nonlinear potential term.

2. NONLINEAR TERM IN NON-ABELIAN GAUGE THEORIES

Our basic attention here is devoted to the non-Abelian gauge field SU(2) (for the simplicity we will consider only this gauge group). The Lagrangian is

\[ \mathcal{L} = \frac{1}{4g^2} F_{\mu\nu}^a F^{a\mu\nu}, \]  

where \( F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g\epsilon^{abc} A_\mu^b A_\nu^c \) is the field strength and \( A_\mu^a \) is the gauge potential; \( g \) is the coupling constant; \( \epsilon^{abc} \) are the structural constants of the gauge group SU(2); \( a = 1, 2, 3 \); in this section we use the Einstein summation convention for repeated indices. In the quantum case we have the operators \( \hat{A}_\mu^a \) and \( \hat{F}_{\mu\nu}^a = \partial_\mu \hat{A}_\nu^a - \partial_\nu \hat{A}_\mu^a + g\epsilon^{abc} \hat{A}_\mu^b \hat{A}_\nu^c \). Let us underline that all operators considered here are the operators of interacting fields in contrast with the perturbative techniques where these operators describe non-interacting fields.

Let us consider the nonlinear part of the field strength operator

\[ \hat{F}_{\mu\nu}^a : (\hat{F}_{nl})_{\mu\nu}^a = \epsilon^{abc} \hat{A}_\mu^b \hat{A}_\nu^c. \]  

At first we assume that this product do not