Grammar Theory Based on Quantum Logic

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Motivated by Ying's work on automata theory based on quantum logic (Ying, M. S. (2000). International Journal of Therotical Physics, 39(4): 985–996; 39(11): 2545–2557) and inspired by the close relationship between the automata theory and the theory of formal grammars, we have established a basic framework of grammar theory on quantum logic and shown that the set of \( l \)-valued quantum regular languages generated by \( l \)-valued quantum regular grammars coincides with the set of \( l \)-valued quantum languages recognized by \( l \)-valued quantum automata.

KEY WORDS: quantum logic; quantum automata; quantum grammar.

1. PRELIMINARIES

To provide a new model of quantum computation, Ying used the semantically analysis approach to study the automata theory based on quantum logic. Ying presented a basic framework of automata theory on quantum logic (Ying, 2000a,b). In particular, Ying introduced the orthomodular lattice-valued quantum predicate of recognizability and established some of its fundamental properties. The most interesting result obtained is the Proposition 2 in Ying (2000b) that says that the language recognized by the product of automata is the intersection of the languages recognized by the factors iff the truth-value lattice of the underlying logic is distributive. But an orthomodular lattice possessing distributivity is a Boolean algebra! This negative result may help us to clarify the boundary between classical computation and quantum computation. Lu and Zheng (2002) defined and studied three different types of lattice-valued finite state quantum automata (LQA) and four different kinds of LQA operation, discussed their advantages, disadvantages, and various properties. The most interesting results (Lu and Zheng, 2002) obtained are the Theorem 3.14, Theorem 3.15, and Theorem 3.16 that say that the validity of many properties of the lattice \( LAT (l, \Sigma, \Theta) \), such as whether it is complete,
distributive, or modular, depends on the corresponding properties of the original lattice.

With the close relationship between automaton theory and the theory of formal grammars in our minds, we have to consider whether or not we can establish a grammar theory based on quantum logic corresponding to the automaton theory based on quantum logic established by Ying (2000a,b). If we can do so, could we obtain the relation between quantum automata and quantum grammars corresponding to the classical one?

First let’s review the classical automata theory and formal grammar theory.

1.1. Classical Automaton Theory

Definition 1.1. A finite state automaton is a quintuple $M$ (Howie, 1991), where

$$M = (Q, A, \varphi, i, T)$$

$Q$ is a finite nonempty set, called the states of $M$;
$A$ is a finite nonempty set, called the set of inputs or the alphabet of $M$;
i $i \in Q$ is the initial state of $M$;
$T$ is a nonempty subset of $Q$ and the elements of $T$ are called the terminal states of $M$; $\varphi$ is a mapping from $Q \times A$ to $Q$, called the state transition function of $M$. It is natural to expand $\varphi$ to be a mapping from $Q \times A^*$ to $Q$ in a recursive way by stipulating that

$$\varphi(q, 1) = q(q \in Q) \text{ (1 stands for the empty word)}$$

$$\varphi(q, wa) = \varphi(\varphi(q, w), a)(q \in Q, w \in A^*, a \in A)$$

An element $w$ of $A^*$ is said to recognized by $M$ if $\varphi(i, w) \in T$. The language $L(M)$ recognized by $M$ is the set of all elements $w$ in $A^*$ that are recognized by $M$, that is to say

$$L(M) = \{w \in A^* | \varphi(i, w) \in T\}$$

Let $q(a_1a_2 \ldots a_n) = q'$, then the states $q, q'$ in the automaton are connected by a path

$$q \xrightarrow{a_1} q_2 \xrightarrow{a_2} q_3 \rightarrow \cdots \rightarrow q_n \xrightarrow{a_n} q'$$

The word $a_1a_2 \ldots a_n$ is called the label of the path. A path will be called successful if it begins with the initial $i$ and ends with a terminal state $t$ in $T$. Thus $w \in L(M)$ if and only if there exists a successful path with label $w$. 