Abstract. *M-reps* (formerly called DSLs) are a multiscale medial means for modeling and rendering 3D solid geometry. They are particularly well suited to model anatomic objects and in particular to capture prior geometric information effectively in deformable models segmentation approaches. The representation is based on *figural models*, which define objects at coarse scale by a hierarchy of figures—each figure generally a slab representing a solid region and its boundary simultaneously. This paper focuses on the use of single figure models to segment objects of relatively simple structure.

A single figure is a sheet of medial atoms, which is interpolated from the model formed by a net, i.e., a mesh or chain, of medial atoms (hence the name *m-reps*), each atom modeling a solid region via not only a position and a width but also a local figural frame giving figural directions and an object angle between opposing, corresponding positions on the boundary implied by the *m-rep*. The special capability of an *m-rep* is to provide spatial and orientational correspondence between an object in two different states of deformation. This ability is central to effective measurement of both geometric typicality and geometry to image match, the two terms of the objective function optimized in segmentation by deformable models. The other ability of *m-reps* central to effective segmentation is their ability to support segmentation at multiple levels of scale, with successively finer precision. Objects modeled by single figures are segmented first by a similarity transform augmented by object elongation, then by adjustment of each medial atom, and finally by displacing a dense sampling of the *m-rep* implied boundary. While these models and approaches also exist in 2D, we focus on 3D objects.

The segmentation of the kidney from CT and the hippocampus from MRI serve as the major examples in this paper. The accuracy of segmentation as compared to manual, slice-by-slice segmentation is reported.

Keywords: segmentation, medial, deformable model, object, shape, medical image

1. Introduction

Segmentation via deformable models has shown the advantage of allowing the expected geometric conformation of objects to be expressed (Cootes, 1993; Staub, 1996; Delingette, 1999; among others, also see McInerny, 1996 for a survey of active surfaces methods). The basic formulation is to represent an object by a set of geometric primitives and to deform the object by changing the values of the primitives to optimize an objective function including a match of the deformed object to the image data. Either the objective function also includes a term reflecting the geometric typicality of the deformed object, or the deformation is constrained to objects with adequate geometric typicality. In our work the objective function is the sum of a geometric typicality term and a geometry to image match term.

The most common geometric representation in the literature of segmentation by deformable models has been a mesh of boundary locations. The hypothesis described and tested in this paper is that improved
One of the advantages of a medial representation is that it allows one to distinguish object deformations into along-object deviations, namely elongations and bendings, and across-object deviations, namely bulgings and attachment of protrusions or indentations. An additional advantage is that distances, and thus spatial tolerances, can be expressed as a fraction of medial width. These properties allow positions and orientations to be followed through deformations of elongation, widening, or bending. Because geometric typicality requires comparison of corresponding positions of an object before and after deformation and because geometry to image match requires comparison of intensities at corresponding positions, this ability to provide what we call a *figural coordinate system* is advantageous in segmentation by deformable models.

Medial representations divide a multi-object complex into objects and objects into *figures*, i.e., slabs with an unbranching medial locus (see Fig. 1). In the following we also show how they naturally divide figures into figural sections, and how by implying a boundary they aid in dividing the boundaries of these figural sections into smaller boundary tiles. This natural subdivision into the very units of medical interest provides the opportunity for segmentation at multiple levels of scale, from large scale to small, that provides at each scale a segmentation that is of smaller tolerance than the previous, just larger scale. Such a hierarchical approach was promulgated by Grenander (1981). Such a multi-scale-level approach is required for a segmentation that operates in time linear in the number of the smallest scale geometric elements, here the boundary tiles. The fact that at each level the units are geometrically related to the units of relatively uniform tissue properties yields effective and efficient segmentations.

Our *m-reps* representation described in Pizer (1999) and Joshi (2001) (in the first reference called DSLs) reverses the notion of medial relations descended from Blum (1967) from a boundary implying a medial description to a mesh of medial atoms implying boundaries, i.e., from an unstable to a stable relation. The radius-proportional ruler and the need to have locality at the scale of the figural section require it to use a width-proportional sampling of the medial surface in place of a continuous medial sheet. These latter properties follow from the desire directly to represent shape, i.e., object geometry of some locality that is similarity transform invariant. The specifics are given later in this section.

*M-reps* also extend the medial description to the inclusion of a width-proportional tolerance, providing opportunities for stages of the representation with successively smaller tolerances. Representations with large tolerance can ignore detail and focus on gross...