Geometry and Destiny

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The recognition that the cosmological constant may be non-zero forces us to re-evaluate standard notions about the connection between geometry and the fate of our Universe. An open Universe can recollapse, and a closed Universe can expand forever. As a corollary, we point out that there is no set of cosmological observations we can perform that will unambiguously allow us to determine what the ultimate destiny of the Universe will be.

The traditional philosophy of General Relativity is that Geometry is Destiny. We teach undergraduates that the Universe can exist in one of three different geometries, open, closed and flat, and that once we determine which describes our Universe, this fixes its fate.

In the past few years, however, several features of conventional wisdom in cosmology have fallen by the wayside. By 1995 it was already clear that fundamental observables, from the age of the Universe, to the baryon content, and the nature of large-scale structure, all independently pointed to the possible existence of a non-zero cosmological constant [1]. At the

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very least, there is now definitive evidence that matter, be it dark or luminous, is not sufficiently abundant to result in a flat Universe today [2]. If we are to believe one of the generic predictions of inflation — that we live in an almost exactly flat Universe — a cosmological constant, or some form of energy very much like it is the only way out.

This speculation received dramatic support a year ago, with independent claims by two groups that Type 1a supernova, when used as standard candles, indicated that the expansion of the Universe is accelerating [3,4]. The simplest explanation of this result is the presence of a cosmological constant.

Most recently, observations of the Doppler peak in the Cosmic Microwave Background anisotropies have begun to provide more definitive evidence that we live in a flat Universe today [5]. When this fact is combined with the SN 1a data, and the data from large-scale clustering, a parameter range of $\Omega_M \approx 0.3 - 0.4$ and $\Omega_\Lambda \approx 0.6 - 0.7$ appears to be strongly favored [6].

While it is premature to claim, on the basis of the existing data, that a $\Lambda$-dominated flat model actually describes our Universe, it is not premature to explore its possible ramifications. Recently, for example, an analysis has been performed that suggests that this observation will have important implications for the future of life in our Universe [7]. Here we focus on a more general feature associated with the incorporation of a cosmological constant into our models: The one-to-one correspondence between geometry and evolution is forever lost.

The mathematical basis of this is described simply. Einstein’s equations imply, for an isotropic and homogeneous Universe, the following evolution equations for the cosmic scale factor, $R(t)$:

\[
H^2 \equiv \left( \frac{\dot{R}}{R} \right)^2 = \frac{8\pi G}{3} \frac{\rho_{TOT}}{R^2} - \frac{k}{R^2}, \tag{1}
\]

\[
\frac{\dot{R}}{R} = -\frac{4\pi G}{3} \sum_i \rho_i (1 + 3w_i), \tag{2}
\]

where $k$ is the signature of the 3-curvature, the pressure in component $i$ is related to the energy density by $p_i = w_i \rho_i$ and the total energy density $\rho_{TOT} = \sum_i \rho_i$. The evolution of the energy density in component $i$ is determined by

\[
\frac{d\rho_i}{\rho_i} = -3(1 + w_i) \frac{dR}{R} \quad \Rightarrow \quad \rho_i \propto R^{-3(1 + w_i)}. \tag{3}
\]