Compactification, Vacuum Energy and Quintessence †

M. C. Bento¹ and O. Bertolami¹

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We study the possibility that the vacuum energy density of scalar and internal-space gauge fields arising from the process of dimensional reduction of higher dimensional gravity theories plays the role of quintessence. We show that, for the multidimensional Einstein-Y ang-Mills system compactified on a $R \times S^3 \times S^d$ topology, there are classically stable solutions such that the observed accelerated expansion of the Universe at present can be accounted for without upsetting structure formation scenarios or violating observational bounds on the vacuum energy density.

KEY WORDS: Dimensional reduction; vacuum energy and accelerated expansion

1. INTRODUCTION

Recently, strong evidence has been emerging that the Universe is dominated by a smooth component with an effective negative pressure and expanding in an accelerated fashion. These findings arise from the study of more than 50 recently discovered Type IA Supernovae with red-shifts greater than $z \geq 0.35$ [1]. Such studies, carried out by two different groups
[1,2], lead to the striking result that the deceleration parameter

$$q_0 \equiv -\frac{\ddot{a}}{a} a,$$

(1)

where $a(t)$ is the scale factor, is negative,

$$-1 \lesssim q_0 < 0.$$

(2)

It follows from the Friedmann and Raychaudhuri equations for an homogeneous and isotropic geometry that, if the sources driving the expansion are vacuum energy and matter, with equation of state $p = \omega \rho$, $-1 \leq \omega \leq 1$, then the deceleration parameter is given by

$$q_0 = \frac{1}{2} (3\omega + 1) \Omega_M - \Omega_\Lambda,$$

(3)

where $\Omega_M (\nu)$ denotes the energy density of matter (vacuum) in units of the critical density. For a Universe where the matter component is dominated by non-relativistic matter or dust, $\omega = 0$, and therefore the combination $\Omega_M \sim 0.4$ and $\Omega_\Lambda \sim 0.7$ seems observationally favoured. Of course, the value $\Omega_\Lambda \sim 0.7$, although consistent with observation (see Ref. 3 for a list of the important constraints), implies a quite unnatural fine tuning of parameters if it arises from any known particle physics setting (see Ref. 4 for a thorough review and Refs. 3,5 for possible connections with fundamental symmetries like Lorentz invariance and S-duality in string theories). Furthermore, $\Omega_M$ and $\Omega_\Lambda$ of the same order suggests that we live in a rather special cosmological period.

While the most straightforward candidate for a smooth component is a cosmological constant, a plausible alternative is a dynamical vacuum energy, or “quintessence”. Suggestions along these lines have been proposed a long time ago [6], although yielding a vanishing deceleration parameter. A number of quintessence models have been put forward, the most popular of which invoke a scalar field with a very shallow potential, which until recently was overdamped in its evolution by the expansion of the Universe, allowing for its energy density to be smaller than the radiation energy density at early times, such that at present $\Omega_M \lesssim \Omega_\Lambda [7,8]$. It was also shown that scalar fields with an exponential type potential can, under conditions, render a negative $q_0 [9,10]$. Other suggestions include the string theory dilaton together with gaugino condensation [11], an axion with an almost massless quark [12], a time-dependent vacuum energy induced by $D$-particle recoil [13], etc. However interesting, most of these suggestions necessarily involve a quite severe fine tuning of parameters [14]. This fact