Dirac Fields in 3D de Sitter Spacetime

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We show that the Dirac equation is separable in the circularly symmetric metric in three dimensions and when the background spacetime is de Sitter we find exact solutions to the radial equations. Using these results we show that the de Sitter horizon has a cross section equal to zero for the massless Dirac field, as in the case of the scalar field. Also, using the improved brick wall model we calculate the fermionic entropy associated with the de Sitter horizon and we compare it with some results previously published.

KEY WORDS: 3D de Sitter; Dirac field; absorption; brick wall model.

1. INTRODUCTION

The de Sitter spacetime is a simple solution to the Einstein equations with positive cosmological constant, nevertheless, its properties have been object of many investigations. These investigations have been motivated by different reasons, among them we can mention the following: the study of the properties of the quantum fields in curved spacetimes [1], inflation [2], the recently proposed dS/CFT correspondence [3, 4] and the observations of the type Ia supernovas that indicate that our Universe is in an accelerated expansion epoch [5].

Recently Myung and Lee proved that the cosmological horizon of the de Sitter spacetime (in $n \geq 3$ dimensions) [6] does not absorb the scalar field if for its radial radial part we take the solutions of the radial equation that are regular at $r = 0$. Undoubtedly it is convenient to investigate if this interesting result is valid also for fields of spin different from zero. One of the objectives of this work is to investigate if the before mentioned result is valid for the Dirac field propagating on the three-dimensional de Sitter spacetime. In order to do this analysis it is necessary...
to know the exact solutions to the Dirac equation in the three-dimensional de Sitter spacetime that we find in the present paper using the separation of variables method.

As is well known, the de Sitter cosmological horizon has thermodynamical properties similar to those of the black hole horizon. In particular it has associated an entropy, and the origin of this entropy has been investigated using different methods [4, 7–9]. In Ref. [9] Kim studied the entropy of the three-dimensional cosmological horizon using the brick wall model (BWM) [10]. Kim [9] considered a scalar field, nevertheless, it is also possible to use fields with spin different from zero when the BWM is applied in order to calculate the entropy of black holes as in Refs. [11, 12]. Another objective of the present work is to calculate the fermionic entropy of the three-dimensional cosmological horizon using the improved brick wall model (IBWM) [13].

The outline of this paper is as follows. In section 2 we describe some properties of the three-dimensional de Sitter spacetime. In section 3, being motivated by the Newman-Penrose formalism [14], we select a triad that is convenient in order to apply the separation of variables method to the Dirac equation in the circularly symmetric metric in three dimensions and we take the limit of these results when the three-dimensional background spacetime is de Sitter. The potential analysis of the radial equations of the Dirac equation on the de Sitter spacetime, obtained in section 3, is done in section 4. In the next section we find the exact solutions to the radial equations previously obtained when the background spacetime is de Sitter and using these solutions we analyze the radial flux of the Dirac field. In section 6 using the IBWM we calculate the entropy of the cosmological horizon due to the Dirac field. Section 7 contains a brief discussion of the results obtained. Finally in the Appendix we do the separation of variables for the massive Dirac equation and we outline a procedure to obtain exact solutions to its radial equations.

2. 3D DE SITTER SPACETIME

The de Sitter spacetime is the maximally symmetric solution of the Einstein equations with positive cosmological constant in the vacuum. There are numerous coordinate systems that can be used in the discussion of several aspects of the de Sitter spacetime (see, for example, Refs. [15, 16] for some recent reviews of the properties of the de Sitter spacetime).

The metric of the three-dimensional de Sitter spacetime in static coordinates [15, 16] takes the form

$$ds^2 = \left(1 - \frac{r^2}{l^2}\right) dt^2 - \left(1 - \frac{r^2}{l^2}\right)^{-1} dr^2 - r^2 d\theta^2,$$

where the coordinate $\theta$ has period $2\pi$, $0 \leq r \leq l$ and in three dimensions $\Lambda = \frac{1}{l^2}$, with $\Lambda$ standing for the cosmological constant. These coordinates are useful