PLASMA INVESTIGATIONS

Dynamics of a Toroidal Plasma Cluster and Its Interaction with an Obstacle. Interaction of Incident and Reflected Flows

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Abstract—A physical-and-mathematical formulation is given of the problem on the interaction between two opposing (incident and reflected) flows of plasma formed upon collision of a plasma cluster with a target. Expressions are given for determining the rates of momentum and energy transfer upon elastic collisions of particles, and the kinetic processes are investigated which define the charge composition and radiation of plasma clusters. The main features are described that are characteristic of the construction of a computational model for the calculation of the dynamic and ionization-and-optical characteristics of interacting plasma flows. Results of preliminary numerical investigations are given.

INTRODUCTION

Different approximations are used for the description of the interaction of plasma flows, depending on the correlation between the stopping range of ions $\lambda_{ii}$ and the characteristic dimension of plasma flow $L$. With $\lambda_{ii} \ll L$, the continuous medium approximation is valid and, as a rule, one-velocity equations of magnetohydrodynamics may be used for solving the problem. With $\lambda_{ii} \gg L$, the collision interaction of ions is negligibly low, and their behavior must be described using the collisionless Boltzmann equation, i.e., by the set of Maxwell–Vlasov equations [1, 2],

$$\frac{\partial f_\alpha}{\partial t} + V \cdot \frac{\partial f_\alpha}{\partial r} + \frac{e_\alpha}{m_\alpha} \cdot (E + \frac{V \times H}{c}) \cdot \frac{\partial f_\alpha}{\partial V} = 0,$$

$$\text{rot} E = -\frac{\lambda}{c} \cdot \frac{\partial H}{\partial t}, \quad \text{rot} H = \frac{4\pi}{c} j + \frac{1}{c} \cdot \frac{\partial E}{\partial t},$$

$$\text{div} E = 4\pi n_e, \quad \text{div} H = 0,$$

where the subscript $\alpha = 1, 2$ corresponds to the ions of two interpenetrating plasma flows. The rest of notation is conventional. The continuous medium approximation is usually valid for an electron fluid, because the thermalization of the directional energy of electrons is faster than that of ions, i.e., $\tau_{ee} \sim \sqrt{m/M} \tau_{ii}$.

In addition, in a wide range of experimental conditions, electron flows may be taken to be subsonic, because the ion-electron beam instability leads to a fast heating of electrons to a temperature corresponding to the average velocity of thermal motion $\langle v_e^2 \rangle \sim u_e^2$, where $u_e$ is the velocity of directed motion. The characteristic time of this process is of the order of inverse frequency $\omega_{pe}^{-1}$ and, as is revealed by the numerical investigations of Davidson et al. [3], is well described by the quasilinear theory [4]. This approach is referred to as hybrid approximation which assumes the absence of kinetic, in particular ionization, processes; it cannot be used to solve numerous applied problems, because it fails to provide an answer to the question about important characteristics of plasma such as its degree of ionization and charge composition.

It was demonstrated by the results of a number of calculations [5–7] that, in the case of $\lambda_{ii} \gtrsim L$, a fairly good approximation for describing interpenetrating plasma flows is provided by a multifluid model which takes into account both the momentum and energy transfer in elastic collisions and the entire complex of inelastic kinetic processes defining the charge composition and nonequilibrium radiation of plasma. Satisfactory results are usually produced by a three-fluid
model describing the characteristics of two interpenetrating sorts of ions and of the flow of electrons.

The experimental investigations of the interaction between a flow incident on an obstacle and that reflected from the obstacle in a longitudinal magnetic field made it possible to conclude that, even with \( \lambda_{ii} > L \), the magnetic field effect and the possible development of instability result in a rapid randomization of the transverse component of ion velocity, and the plasma behavior is similar to that of a continuous medium [8–11]. Based on this, Stepanov and Sidnev [12] used the continuous medium approximation to calculate the thermodynamic characteristics and radiation of plasma at rest behind the front of reflected shock wave and estimate the effect of radiation on the overall dynamics of the plasma. Therefore, the value of the magnetic field, which is not included in Eqs. (1)–(7), is calculated in a non-self-consistent formulation, i.e., disregarding the inverse effect of the magnetic field on the plasma. The electric field (significant only in the region of high gradients) was calculated using a special Eulerian grid. The plasma parameters and the entire two-dimensional pattern of interaction of interpenetrating plasma flows were

\[
\frac{\partial n_i}{\partial t} + \text{div}(n_i \cdot \mathbf{V}_i) = S_i, \quad \text{div}(n_e \cdot \mathbf{V}_e) = S_e, \quad (4)
\]

\[
m_e n_e \frac{d\mathbf{V}_e}{dt} = -\nabla P_e - e n_e \cdot \mathbf{E} + \mathbf{R}_e, \quad (5)
\]

\[
\frac{3}{2} n_e \frac{dT_e}{dt} + n_e T_e \text{div} \mathbf{V}_e = \text{div}(\lambda \nabla T_e) + S_e + Q_e. \quad (6)
\]

\[
\text{div} \mathbf{E} = 4\pi e \left[ \sum z(n_{1z} + n_{2z}) - n_e \right]. \quad (7)
\]

Here, \( S_{ii}, S_e, \mathbf{R}_i, \mathbf{R}_e, \mathbf{Q}_i, \) and \( Q_e \) denote the rates of variation of the concentration, momentum, and energy of particles, respectively; \( \lambda \) is the coefficient of electron thermal conductivity; the rest of notation is conventional. The solution of this set of equations makes it possible to determine the following characteristics of plasma: \( n_{1z}, n_{2z}, n_e, \mathbf{V}_i, \mathbf{V}_e, T_1, T_2, T_e, \) and \( \mathbf{E} \). The subscript 1 indicates incident flow, and the subscript 2, reflected flow. The following equations of state are valid for each one of the flows: \( P_1 = n_1 T_1, \) \( P_2 = n_2 T_2, P_e = n_e T_e, \) where \( n_1 = \sum_{z} n_{1z}, n_2 = \sum_{z} n_{2z}, \) and \( z = 0, 1, 2, \ldots, z_m. \) For neon, \( z_m = 10. \) For making a distinction between the dynamic and ionization processes, the relative concentrations of ions and electrons were introduced,

\[
\alpha_z = \frac{n_{1z}}{n_1}, \quad \alpha_{2z} = \frac{n_{2z}}{n_2}, \quad \alpha = \frac{n_e}{n}, \quad n = n_1 + n_2. \quad (8)
\]

In so doing, Eq. (1) is divided into two groups,

\[
\frac{\partial n_{iz}}{\partial t} + \text{div}(n_{iz} \cdot \mathbf{V}_i) = 0, \quad \frac{d\alpha_z}{dt} = \frac{S_{iz}}{n_i}, \quad i = 1, 2; \quad \frac{d\alpha}{dt} = \frac{\partial}{\partial t} + (\mathbf{V}_i \nabla). \quad (9)
\]

Note that the results of detailed investigations have demonstrated that the electric and magnetic fields exist only in the region of high gradients and in the vicinity of the target surface. They are localized in a relatively small space and have no significant effect on the overall dynamics of the plasma. Therefore, the value of the magnetic field, which is not included in Eqs. (1)–(7), is calculated in a non-self-consistent formulation, i.e., disregarding the inverse effect of the magnetic field on the plasma. The electric field (significant only in the region of high gradients) was calculated using a special Eulerian grid. The plasma parameters and the entire two-dimensional pattern of interaction of interpenetrating plasma flows were