CALCULATING THE CRITICAL PARAMETERS CHARACTERIZING THE THERMAL INSTABILITY OF A VISCOELASTIC PRISM WITH A STRESS CONCENTRATOR UNDER HARMONIC COMPRESSION

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The paper proposes simplified schemes for analyzing critical thermal states of a notched rectangular viscoelastic prism under harmonic loading. The influence of the problem parameters on the critical stress amplitude is analyzed. The notch does not decrease the critical stress. Varying the problem parameters hardly affects the critical level of temperature. This level corresponds to the glass-transition temperature of an amorphous polymer.

Keywords: vibrational heating, thermal instability, viscoelastic prism, stress concentration

Studying vibrations and vibrational heating of thermoplastic materials, in particular metals [16, 19, 20] and polymers [13, 15] is of theoretical and applied importance.

It is well known that mechanical loss is high in regions of structural changes, such as glass-transition and plastic regions, in many thermoplastic polymeric materials [11]. The heating temperature in these regions abruptly increases under cyclic loading. Therefore, certain cyclic-loading and heat-transfer conditions may lead to thermal fatigue failure due to material softening or even melting, rather than ordinary mechanical fatigue failure via cracking [6]. This problem is important for many fields of engineering and technology that use thermoplastic materials.

The first studies on vibrational heating in polymeric (mainly elastomer) materials date back to the 50s of the last century. The conceptual understanding of the process was given in [1, 12]. The results of later studies were reported on in [3, 5, 15]. Such studies are very important for the problem of fatigue failure of thermoplastic materials. The current state of the art in the experimental research and theoretical simulation of thermal fatigue failure is analyzed in [7, 10].

Thermal instability is the most dangerous scenario of thermal fatigue failure. It is characterized by avalanche-like increase in temperature. Basic studies on the subject are due to Barenblatt et al. and Schapery [1, 12]. They demonstrated that thermal instability might occur if the load parameters exceeded some critical levels. The critical parameters were calculated in [4, 6].

Thermal instability analysis implies solving two basic problems. One is to determine the critical parameters that trigger avalanche-like heating, which allows evaluating the high-cycle fatigue life of structural members. The other problem is to study the kinetics of nonstationary (postcritical) heating up to thermal failure, which allows evaluating the life of articles under intensive postcritical loading.

Another area of application of the thermal-instability problem is ultrasonic technologies, in particular ultrasonic welding [2]. Here, postcritical heating is a desired mode. It provides the minimum duration of the thermal cycle and high efficiency of the technological process.

Mechanical failure usually sets in at stress concentrators. It is they that become centers of thermal fatigue failure. However, there are very few published experimental data on the thermal state of bodies with stress concentrators such as notches [7]. Numerical results were unavailable until recently. The authors of [13, 16] studied the vibrations and vibrational heating of a...
notched rectangular prism and discovered a local zone of heating, up to melting, which forms at the vertex of the notch and extends deep into the body. As established in [13], this process is loss of thermal stability in a thermally local mode. It is similar to adiabatic thermal shear bands in plastic materials under high-speed loading [17].

The present paper sets out to analyze the influence of the physical and geometric characteristics of a rectangular prism with a notch on the thermal instability boundary.

1. Problem Statement. In a rectangular Cartesian coordinate system $xOy$, the plane thermostrained state of a linear viscoelastic prism with a notch (Fig. 1) $|y|<L, 0<x<H$, under harmonic compression is described by the vibration and heat-conduction equations

\[
\text{div } \bar{\sigma} + \rho \omega^2 \bar{u} = 0, \quad (1)
\]

\[
\bar{\epsilon} = \nabla (k \nabla \theta) + D', \quad (2)
\]

the constitutive equations for stresses and dissipation rate

\[
\bar{s} = 2\bar{G} \bar{\epsilon}, \quad \bar{\sigma}_{kk} = 3\bar{K} \bar{\epsilon}_{kk}, \quad D' = \frac{\omega}{2} \text{Im}(\bar{\sigma} : \bar{\epsilon}^*), \quad \bar{\epsilon}_{zz} = 0,
\]

and the boundary and initial (for temperature) conditions

\[
\bar{\sigma}_{yy} = -i\sigma_0, \quad \bar{\sigma}_{xy} = 0, \quad y = \pm L, \quad \bar{\sigma}_{xx} = \bar{\sigma}_{yy} = 0, \quad x = 0, H,
\]

\[
-k\nabla \theta = \alpha (\theta' - \theta_c) \quad \text{on } l, \quad \theta(x, y, 0) = \theta_0, \quad (3)
\]

where $\bar{\sigma}, \bar{\epsilon}, \bar{s}, \bar{\epsilon}$, and $\bar{u}$ are the complex amplitudes of the stress and strain tensors, their deviators, and displacement vector; $\theta$ and $D'$ are the period-average temperature and dissipation rate function; $\bar{G}$ and $\bar{K}$ are the complex shear and bulk moduli; $\alpha$ is the heat-transfer factor; $\omega$ is the frequency of vibrations; $\rho$ is density; $c$ and $k$ are the specific heat capacity and heat conductivity coefficient; $\theta_0$ and $\theta_c$ are the initial temperature and ambient temperature; $l$ is the cross-sectional boundary of the prism; $\bar{\sigma} : \bar{\epsilon}^*$ is the convolution of tensors; and $\bar{\epsilon}^*$ is a complex-conjugate tensor.

If the temperature field is stationary, then problem (2)–(3) reduces to

\[
\nabla (\bar{G} \nabla \theta) + \eta \Psi(x, \theta) = 0, \quad (4)
\]

\[
\nabla \theta \cdot \bar{n} + \bar{\chi}(\theta)(\theta' - \theta_c) = 0.
\]