Refinement on the theoretical analysis of multifer fiber ceramic capacitor

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In a previous paper, a new type of ceramic capacitor, multifer fiber ceramic capacitor (MFC), was
designed and analyzed to compare the properties with multilayer ceramic capacitors (MLC).
Refinement on the theoretical analysis of the MFC is presented in this paper for the capacitors
made from dielectric film less than 2 μm thick. A critical λc exists above which the specific
capacitance of an MFC is higher than an MLC, where λ is the ratio of the dielectric thickness of
the MLC and the MFC. Such a λc value is readily attainable because a high-quality dielectric
coating on fiber substrates can be easily produced by modern thin-film technology. In other
words, MFC has the potential to surpass MLC in term of the specific capacitance. Core fibers
(inner electrodes) with a small diameter should be selected, whenever possible, for improved
specific capacitance. The choice of possible materials for the MFC is also discussed.

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1. Introduction
In a previous paper [1], a new type of ceramic capacitor, multifer fiber ceramic capacitor (MFC) was designed and the possible techniques for the MFC preparation were also proposed. An MFC consists of multiple single-fiber capacitors (elements) connected in parallel, each element containing a core fiber electrode, a dielectric coating, and an outer electrode. The individual elements are combined with a conductive or non-conductive matrix to form an MFC. Recently, important progress has taken place in the commercial production of multilayer ceramic capacitors (MLC). Printing and adhesive-film techniques have been introduced to reduce the thickness of the dielectric film [2]. By the use of these new techniques, dielectric films as thin as 2 μm have become possible. By contrast, the conventional slip casting limits the dielectric thickness to about 10 μm. Our previous analysis was based on the assumption that the thickness of the electrodes was much smaller than that of the dielectric, and hence negligible. However, the validity of this assumption is in question for the most advanced capacitors. This new development raises a question whether the MFC is still competitive against the new generation of MLC in terms of their specific capacitance. The major objective of this paper is to answer this question. The readers are advised to consult with the previous paper for the background information.

2. Fundamentals of multifer fiber ceramic capacitor
A summary of the previous paper is present below, but the difference is that the electrode thickness has been taken into consideration. The capacitance (C) of an MLC is given by the following expression:

\[ C_L = \sum_{i=1}^{n_L} C_i = n_L \varepsilon \frac{A}{t_L} = \frac{d}{t_L + t_R} \varepsilon \frac{A}{t_L} \]

\[ = \frac{d^2}{(t_L + t_R)^2} \varepsilon \frac{A}{t_L} \]  

(1)

where \( n_L \), \( t_L \), and \( A \) denote the number, thickness, and area, respectively, of the dielectric layers; \( d \) and \( t_R \) stand for the capacitor thickness and the electrode thickness. Clearly the reduction in the thickness of the dielectric layers is the most effective way to increase the capacitance.

The capacitance of an MFC is expressed by:

\[ C_F = \sum_{i=1}^{n_F} C_i \]

\[ = \frac{2\pi d^2}{(D_0 + 2t + 2t_R)^2} \ln((D_0 + 2t + 2t_R)/D_0) \varepsilon \frac{A}{d} \]  

(2)

where \( D_0 \), \( t \), and \( t_R \) are the diameter of the core electrode, dielectric thickness, and electrode thickness, respectively. As in MLC, the capacitance is inversely proportional to the thickness of the dielectric coating.

At the same geometrical volume, the relative capacitance of MFC and MLC is given by:

\[ \alpha = \frac{C_F}{C_L} = \frac{2\pi t_L (t_L + t_R)}{(D_0 + 2t + 2t_R)^2} \ln((D_0 + 2t + 2t_R)/D_0) \]  

(3)

Obviously \( \alpha \) is the ratio of the specific capacitance of MFC and MLC. Assume \( t_L = \kappa t \), then:

\[
\alpha = \frac{2\pi \kappa \kappa^2 \tau^2}{(D_0 + 2\tau + 2\tau^2)^2 \ln\left((D_0 + 2\tau + 2\tau^2)/D_0\right)}
\]  

(4)

In this paper, two situations are considered, one is that the dielectric thickness is much greater than the electrode thickness, \(t_i \gg \epsilon_c\) and \(t \gg \epsilon_c^2\); the other is that the dielectric thickness is comparable with the electrode thickness, \(t_i \sim \epsilon_c^2\) and \(t \sim \epsilon_c^2\).

2.1. \(t_i \gg \epsilon_c^2\) and \(t \gg \epsilon_c^2\)

The thickness of the dielectric layers in an MLC fabricated by the tape-casting technique is normally larger than 10 mm, while the electrode thickness is about 1 \(\mu m\). Therefore, the electrode thickness can be neglected and expression (4) is reduced to:

\[
\alpha = \frac{2\pi \kappa^2 \tau^2}{(D_0 + 2\tau)^2 \ln\left((D_0 + 2\tau)/D_0\right)}
\]  

(5)

For a given \(D_0\) \(\alpha\) attains the maximum value \(\alpha_{\text{max}}\) when:

\[
\frac{\partial \alpha}{\partial t} \bigg|_{\alpha = \alpha_{\text{max}}} = 0
\]  

(6)

One obtains:

\[
\frac{t_m}{D_0} = 1.26
\]  

(7)

and

\[
\alpha_{\text{max}} = 0.64\kappa^2
\]  

(8)

where \(t_m\) is the dielectric thickness of an MFC element at which the MFC has the optimum capacitance compared to the corresponding MLC. For an MFC to have the same capacitance as an MLC, i.e., \(\alpha_{\text{max}} = 1\), \(\kappa = 1.25\) is required. In other words, when the dielectric thickness of an MFC element is 80% of an MLC element, they have the same specific capacitance. The specific capacitance of an MFC is higher than that of an MFC for a thinner dielectric coating.

2.2. \(t_i \sim \epsilon_c^2\) and \(t \sim \epsilon_c^2\)

Although an MFC has the highest specific capacitance against an MLC when the dielectric thickness and the core electrode diameter of the MFC elements are perfectly matched, the combinations are limited by the availability of suitable fibers. The lower limit in diameter of the commercially available fibers is about 5 mm. According to Equations 7 and 8, when a 5-mm core electrode is coated with 6.3 mm of dielectric, the resultant MFC has the same specific capacitance as an MLC with 7.9 mm of dielectric. The advanced printing and adhesive-film techniques can readily achieve this thickness, so that the merits of MFC over MLC are not obvious.

Minimization of electronic components has resulted in the continuous reduction of dielectric thickness in ceramic capacitors, and this trend will continue in the foreseeable future. Therefore, the question that must be answered is whether the MFC still compares favorably to MLC when the dielectric thickness of an MFC element is much smaller than the \(t_m\). The properties of MFC and MLC are to be compared below for the dielectric thickness less than 2 \(\mu m\).

Depending on the processing techniques used, the thickness of electrodes may range from 0.05 to 1 \(\mu m\). When the thickness of a conductive film is less than 0.05 \(\mu m\), its conductivity drops dramatically and eventually turns insulating [3]. Therefore, films thinner than 0.05 \(\mu m\) are not considered useful as electrodes. For a dielectric layer less than 2 \(\mu m\), the thickness of electrodes can no longer be neglected. For the convenience of the analysis, we assume that the electrode thickness is half of the dielectric thickness i.e., \(t_i = \frac{1}{2} t_m, t_c = \frac{1}{2} t_i\), so that Equation 4 becomes:

\[
\alpha = \frac{3\pi \kappa^2 \tau^2}{(D_0 + 3\tau)^2 \ln\left((D_0 + 3\tau)/D_0\right)}
\]  

(9)

Table 1 shows the \(\alpha\) values at different dielectric thickness for the core electrodes of 20 \(\mu m\). For example, the capacitance of the MFC with a dielectric thickness of 0.1 \(\mu m\) is 1.535 times that of the MLC containing 1 \(\mu m\) dielectric layers (\(\kappa = 10\)).

If the \(\kappa\) value at \(\alpha = 1\), where the specific capacitance of the MFC is the same as that of the MLC, is nominated as the critical dielectric thickness ratio and denoted by \(\kappa_c\), then \(\kappa_c\) can be derived from Equation 9 as:

\[
\kappa_c = \frac{1}{\sqrt{3\pi}} \cdot \frac{D_0 + 3t_c}{t_c} \cdot \sqrt{\ln\left(\frac{D_0 + 3t_c}{D_0}\right)}
\]  

(10)

where \(t_c\) is the critical dielectric thickness of an MFC. This relation is illustrated in Fig. 1.

The curve divides the graph into two regions. Above the line or at \(\kappa > \kappa_c\), the specific capacitance of an MFC is higher than an MLC, while the trend is reversed below the line. From Equation 9, \(\alpha\) is proportional to \(\kappa^2\), so that a higher \(\kappa\) is highly desirable for an MFC to outperform an MLC. Now the obvious question is whether the dielectric thickness of an MFC can be sufficiently small to satisfy the relation \(\kappa > \kappa_c\). The thickness of dielectric films is mainly determined by the processing methods available. For MLC, a green film as low as 2 \(\mu m\) is obtainable by the printing and adhesive-film techniques, which is reduced to about 1.5 \(\mu m\) after sintering. The ultimate lower limit of these techniques is believed to be 1 \(\mu m\) [4]. In Equation 9, let \(\kappa_c \cdot t_c = 1\), the solution is \(t_c = 0.1519\mu m\) and \(\kappa_c = 6.579\) (\(D_0 = 20\mu m\)). In other words, as long as the dielectric thickness of the MFC elements is less than 0.1519 \(\mu m\), the MFC would have a specific capacitance insurmountable by the MLC made with the present commercial methods. Such a thickness

<table>
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<th>1.5</th>
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