RESONANT CONFLUENCE OF SINGULAR POINTS
AND STOKES PHENOMENA

A. A. GLUTSYUK

Abstract. We consider a linear ordinary differential equation with resonant irregular singularity of generic type. For its generic deformation that splits the irregular singularity of the unperturbed equation into Fuchsian singularities of the perturbed one, the nonformal analytic classification invariants (Stokes operators) of the unperturbed equation are expressed via limit transition operators that compare appropriate monodromy eigenbases of the perturbed equation. We do this for all values of the Poincaré rank and the dimension (denoted by $k$ and $n$ respectively), except for the case where $k = 1$ and $n \geq 3$. We show (Theorems 2.1 and 2.2 in Sec. 2) that appropriate branches of the monodromy eigenfunctions of the perturbed equation converge to appropriate canonical solutions of the unperturbed equation. In the case where $k = n = 2$, this statement implies that the Stokes operators are limits of transition operators between appropriate monodromy eigenbases of the perturbed equation (Corollary 2.1). We give a generalization of the last statement for the case of higher Poincaré rank and dimension (Corollary 2.2).

1. Introduction

1.1. Brief statements of results, the plan of the paper and the history.

Definition 1.1. Consider a linear ordinary differential equation

$$\dot{z} = \frac{A(t)}{t^{k+1}} z, \quad z \in \mathbb{C}^n, \quad t \in \mathbb{C}, \quad |t| < 1, \quad k, n \in \mathbb{N}, \quad n \geq 2,$$

(1.1)

where $A(t)$ is a holomorphic matrix function. We say that Eq. (1.1) has a nonresonant irregular singular point at $0$ (or, briefly, (1.1) is a nonresonant equation) if the matrix $A(0)$ has distinct eigenvalues. We say that $0$ is a generic Jordan resonant singularity (or, briefly, (1.1) is a generic (Jordan)

2000 Mathematics Subject Classification. 34M35 (34M40).

Key words and phrases. Linear ordinary differential equations with complex time, resonant irregular singularity, Stokes operators, Fuchsian singularity, monodromy, confluence.

1079-2724/04/0400-0253/0 © 2004 Plenum Publishing Corporation
resonant equation) if the matrix $A(0)$ is conjugated to a Jordan cell (let $\lambda$ be its eigenvalue) and

$$(\det(A - \lambda \text{Id}))'(0) \neq 0. \quad (1.2)$$

In both cases the number $k$ is said to be the Poincaré rank (order) of the singular point.

In the paper we consider generic Jordan resonant equation (1.1) with arbitrary values of $k \geq 1$ and $n \geq 2$, except for the case where $k = 1$ and $n \geq 3$. For its generic deformation that splits the irregular singularity 0 of the unperturbed equation into Fuchsian singularities of the perturbed one, the nonformal analytic classification invariants (Stokes operators) of the unperturbed equation are expressed via limit transition operators that compare appropriate monodromy eigenbases of the perturbed equation. We show (Theorems 2.1 and 2.2 in Sec. 2) that appropriate branches of the monodromy eigenfunctions of the perturbed equation converge to appropriate canonical solutions of the unperturbed equation. In the case where $n = k = 2$, this statement implies that the Stokes operators are limits of transition operators between appropriate monodromy eigenbases of the perturbed equation (Corollary 2.1). We give a generalization of this statement for higher Poincaré rank and dimension (Corollary 2.2).

We recall the definition of canonical solutions and Stokes operators of generic Jordan resonant equation (1.1) below in Sec. 1.3.

It is well known (see [9] and Sec. 1.3 below) that any generic Jordan resonant equation (1.1) can be transformed into a nonresonant equation by a (noninvertible) change of variables including the time (relation (1.7) in Sec. 1.3). The nonresonant equation has canonical sectorial solutions and Stokes operators (defined in Sec. 1.2). The similar objects of the Jordan resonant equation (1.1) correspond to those of the nonresonant equation via the inverse variable change.

Earlier in 1919, R. Garnier [4] had studied some particular deformations of some class of linear equations with nonresonant irregular singularity. He obtained analytic classification invariants for these equations by studying their deformations. The complete system of analytic classification invariants (Stokes operators and formal normal form) for general nonresonant irregular singularities of linear differential equations was obtained in [2, 8, 13]. The complete system of analytic classification invariants of generic resonant linear equation was obtained in [9]. The conjecture that Stokes operators of a linear equation with an irregular singular point can be obtained from limit monodromy data of its deformation with Fuchsian singularities was stated by V. I. Arnold in 1984 and a little later independently by J.-P. Ramis (1988), who considered the classical confluenting family of hypergeometric equations and proved the convergence of appropriate branches of monodromy eigenfunctions of the perturbed equation to canonical solutions of