Scaling Laws of Vortex Reconnections

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The vortex reconnection rate $f$ plays an important role in the dynamics of a tangle of quantised vortices in superfluid turbulence. The question which we address is how $f$ depends on the vortex line density $L$. In the case of a homogeneous isotropic tangle of vortices we show that, besides the known regime which scales as $f \sim L^{5/2}$, another regime exists which scales as $f \sim L^2$. In the case of a polarised vortex configuration, we argue that the scaling law changes and show numerical evidence for it. Finally we construct an idealised model of turbulence decay based on vortex reconnections and discuss its implications.

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1. INTRODUCTION

Quantised vorticity occurs in superfluid helium ($^4\text{He}$ and $^3\text{He}$) and in atomic Bose–Einstein condensates. Depending on the particular situation, quantised vortices form an ordered lattice or a turbulent tangle. This article is concerned with turbulent tangles. Turbulent tangles have been studied in helium II and in $^3\text{He}$-B. During a tangle’s evolution, if two vortex filaments come sufficiently close to each other, a vortex reconnection takes place, as shown in Fig. 1. Our aim is to reconsider this process, paying particular attention to the vortex reconnection rate $f$, defined as the number of vortex reconnections per unit time per unit volume, and how it depends on the vortex line density $L$, defined as the length of vortex lines per unit volume.

The plan of the paper is the following. In Section 2 we review the necessary background of vortex dynamics. In Section 3 we discuss the physical significance of $f$. In Section 4 we consider a homogeneous isotropic tangle, determine the scaling law $f \sim L^7$ and show that, besides the known regime
Fig. 1. Schematic vortex configuration (a) before the reconnection and (b) after.

characterised by $\gamma = 5/2$, there is another regime with $\gamma = 2$. In Section 5 we show numerical evidence for the new scaling $\gamma = 2$ and preliminary evidence which suggests that polarisation alters the scaling law. In Section 6 we construct a toy model of turbulent decay which makes explicit the role played by vortex reconnections; then we explore its implications regarding a recent experiment. Section 7 is devoted to conclusions and points to further work.

2. VORTEX DYNAMICS

Observed values of the vortex line density $L$ can be as large$^2$ as $L \approx 10^7$ cm$^{-2}$ in helium II and$^3$ $10^6$ cm$^{-2}$ in $^3$He-B. From $L$ one obtains the typical separation between the vortices, $\delta \approx L^{-1/2}$. Since $\delta$ is many orders of magnitude greater than the vortex core radius $a_0$, which is proportional to the coherence length (approximately $10^{-8}$ cm in helium II and $10^{-6}$ cm in $^3$He-B), it is natural to represent quantised vortices as space curves of equation $s = s(\xi, t)$ where $t$ is time and $\xi$ is arc length. The inertia of the vortex core is negligible, so the motion of a vortex filament is determined by the balance of Magnus and friction forces. Following Schwarz,$^4$ the governing equation of motion is

$$\frac{ds}{dt} = \mathbf{v}_s + \mathbf{v}_i + \alpha \mathbf{s}' \times (\mathbf{v}_{ns} - \mathbf{v}_i) - \alpha' \mathbf{s}' \times (s' \times (\mathbf{v}_{ns} - \mathbf{v}_i)), \quad (1)$$

where $s' = ds/d\xi$, $\alpha$ and $\alpha'$ are temperature dependent friction coefficients,$^5,6$ $\mathbf{v}_n$ is the normal fluid velocity, $\mathbf{v}_s$ is any imposed superflow and $\mathbf{v}_{ns} = \mathbf{v}_n - \mathbf{v}_s$. In helium II $\alpha'$ is smaller than $\alpha$ and is sometimes neglected. The self-induced velocity $\mathbf{v}_i$ of the vortex filament at the point $s$, is given by