MODELING OF CONTACT CONDITIONS UNDER DEFORMATION OF ROCK SAMPLES

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Equations connecting the stress and displacement components on the boundaries of rock sample are obtained. Examples of their numerical realization are cited.

Equations, stresses, displacements, elasticity, system, solution, problem

To study mechanism of rock failure, a series of analytical and experimental investigations is usually required. As a rule, parameters of failure depend on loading conditions and state of contact surfaces. The problems on contact are complicated and nonlinear as a consequence of boundary mobility and friction between contacting surfaces. Physical effects are connected with the fact that slippings can appear on the contact between interacting bodies. In slipping zones, energy accumulated is lost. Therefore, functions describing processes of loading and unload will not be linear, which can change the compressive force, for example.

Ideally, solution of these problems requires complete information on cracks and inclusions, influence of external actions on rock properties as well as experimental data on measurement taken in all accessible parts of the sample. Determination of constants and functions characterizing material deformation is based on the data from mechanical tests and physical investigations into rocks. For block media, these problems are considered in [1 – 3].

This paper presents mathematical modeling of stress-strain state of the rectangular domain Ω (Fig. 1). A system of integral equations is derived to consider all components of boundary values of unknown functions in two-dimensional case for three basic problems of elasticity theory on the contour \( \Gamma = \Gamma_1 + \Gamma_2 + \Gamma_3 + \Gamma_4 \). Two of four stress and displacement components are formulated on statement of boundary problem, the rest are calculated from the system proposed.

![Fig. 1. Calculated scheme for determining stress-strain state of rock sample](image)

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In most interesting cases of testing rock samples, calculating pillar deformation, etc., external forces are transferred to $\Omega$ through the contact with other bodies so that only the resultant force vector applied to $\Gamma_1$ or $\Gamma_3$ is known. Here, it is impossible to formulate strict conditions of interaction in the zone of contact. Usually the simplest variants of boundary conditions are assumed. They badly model the process of loading that is reduced either to perfect slipping or to complete cohesion on assumption of absolute rigidity of external bodies relative to $\Omega$ [4].

In this connection, in situ data are required for making contact conditions within experimental-analytical method more accurate [5]. The same is referred to inverse problems, for example, selection of boundary condition ensuring maximal carrying capacity of the object under modeling.

For the domain $\Omega$ (Fig. 1), formulate boundary conditions in the form of three problems:

on $\Gamma_1$ \[ \tau = \tau_0(x), \quad v = v_0(x), \] \hspace{1cm} (1)

on $\Gamma_3$ \[ \tau = \tau_0(x), \quad v = -v_0(x); \]
on $\Gamma_1$ \[ u = u_0(x), \quad v = v_0(x), \] \hspace{1cm} (2)
on $\Gamma_3$ \[ u = u_0(x), \quad v = -v_0(x); \]
on $\Gamma_1$ \[ \tau = \tau_0(x), \quad \sigma_y = \sigma_{y0}(x), \] \hspace{1cm} (3)
on $\Gamma_3$ \[ \tau = \tau_0(x), \quad \sigma_y = -\sigma_{y0}(x). \]

For each of them, the boundary conditions on $\Gamma_2$ and $\Gamma_4$ are the same: \( \tau = \sigma_x = 0 \). Here, \( \tau, \sigma_x, \) and \( \sigma_y \) are the tangential and normal stresses; \( u \) and \( v \) are the displacement components.

In [6], the system of equations connecting boundary values of all three basic problems of elasticity theory is presented as:

\[
 f(t_0) + 2\mu g(t_0) = -\frac{1}{\pi i} \int_{\Gamma} \frac{f + 2ug}{t - t_0} dt,
\]

\[
 x \tilde{f} - 2\mu \tilde{g} = -\frac{1}{\pi i} \int_{\Gamma} \frac{x \tilde{f} - 2\mu \tilde{g}}{t - t_0} dt - \frac{1}{\pi i} \int_{\Gamma} (f + 2ug) \frac{\tilde{t} - \tilde{t}_0}{t - t_0} dt,
\]

where \( f(t) = i \int_0^t (X_n + iY_n) ds; \quad g = u + iv; \quad X_n \) and \( Y_n \) are the stress components in the direction of \( x \) and \( y \) axes; prime over function denotes a complex-conjugate value; \( i \) is the imaginary unit; \( x = 3 - 4v; \quad \mu = 0.5E/(1+v); \quad v \) is Poisson’s ratio; \( E \) is Young’s modulus; \( t \in \Gamma; \quad t_0 \) is the affix of point of the boundary \( \Gamma \).