Feynman's Ratchet and Pawl

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While many papers in the last few years have dealt with various equations euphemistically called "ratchets," the original Feynman two-temperature setup has been left largely unchallenged. We present here a look at the details of how this famous engine actually generates motion from a temperature difference.

Maxwell understood correctly that the Second Law is certain only in a statistical sense. He attempted to show this(1) by the device now called "Maxwell demon:" a being of molecular size who would sort fast molecules from slow molecules, thus generating a thermal gradient out of an initially isothermal condition. Unfortunately, Maxwell did not realize that the demon would itself be subject to fluctuations of the same type and size as those it was trying to take advantage of. By the 20s and 30s people like Smoluchovsky(2) and Szilard(3) showed that once a system is in thermal contact with a reservoir it does not matter whether it is large or small at the molecular scale: the operations of the demon will always be subject to the Second Law. They started a long tradition(4, 5, 6) of the Maxwell demon as a means to probe the underlying nature of statistical mechanics at the small scale, with the Second Law no longer into question.

But then the nature of the game changed dramatically, and not from within physics. Advances in molecular biology made it clear that cells are populated with molecular machinery operating near the limits of thermal energies, and that these machinery do indeed perform the kinds of tasks usually entrusted to Maxwell demons. We are, in a sense, made of demons.

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We propose here a retreat to the purely conceptual realm, to consider Feynman’s *Ratchet and Pawl* mechanism, just for the fun of understanding some details of irreversible systems subject to multiple temperatures. The picture that emerges is substantially more complicated, but also more rich, than Feynman’s analysis outlined: a story of probability currents circulating in large eddies, shedding small amounts of probability on their borders to make the engine work. We’ll build from discussions of how to generate motion from thermal gradients, as outlined by Landauer, Buttiker and van Kampen, and from detailed analysis of the intrinsic losses incurred when touching two thermal baths simultaneously, as outlined by Parrondo and Español, and by Sekimoto.

We’ll proceed as follows. We’ll set up our equations modeling the ratchet. Then we’ll argue a boundary layer approximation (BLA) that collapses to a case studied by Buttiker, and show how this picture formalizes Feynman’s discussion. We’ll then do numerical simulation, and show that the boundary layer approximation is incorrect because it assumes a single layer: the motion of the system is organized in an elongated roll, with an updraft bottom layer and a downdraft upper layer. While the net difference between top and bottom currents that provides the interwell motion is reasonably described by the BLA, neither the large-scale picture nor the resulting losses are. Furthermore, upon reversal of the temperature difference, a new mechanism arises that never operates in the vicinity of the boundary at all. We’ll wrap up by analyzing these features in analytical detail for a linear system.

1. **RATCHET AND PAWL**

We will not here review Feynman’s setup, lest we might, by doing so, deprive the reader from a perfect excuse to read once more Chapter 46 of the *Lectures on Physics*.

We’ll work in the overdamped regime, since the underdamped system gives essentially similar answers at a much higher cost, and underdamped systems are physically much more difficult to realize. We’ll call \( x \) the degree of freedom associated to the ratchet-axle-vane system (p.b.c., since it’s an angle) and \( y \) the position of the pawl; the shape of the ratchet’s teeth enters as a boundary condition (See Fig. 1). Then,

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\begin{align*}
\dot{x} &= -\partial_x V + \xi_1(t) \\
\dot{y} &= -\partial_y V + \xi_2(t) \\
\langle \xi_1(t) \xi_j(s) \rangle &= 2T_y \delta(t-s)
\end{align*}
\]