Equilibrium, Games, and Pricing in Transportation and Telecommunication Networks

EITAN ALTMAN
INRIA, 2004 Route des Lucioles B.P. 93, 06902 Sophia Antipolis, France
email: altman@sophia.inria.fr

LAURA WYNTER
IBM Watson Research Center, P.O. Box 704, Yorktown Heights, NY 10598, USA
email: lwyster@us.ibm.com

Abstract

Network equilibrium models that have traditionally been used for transportation planning have penetrated in recent years to other scientific fields. These models have recently been introduced in the telecommunication networks literature, as well as in the field of game theory. Researchers in the latter fields are not always aware of the very rich literature on equilibrium models outside of their application area. On the other hand, researchers that have used network equilibrium models in transportation may not be aware of new application areas of their tools. The aim of this paper is to present some central research issues and tools in network equilibria and pricing that could bring closer the three mentioned research communities.

Keywords: Wardrop equilibrium, Nash equilibrium, potential games, variational inequalities, pricing

1. Introduction

Determining the equilibrium state of a traffic network has been a preoccupation of transport planners for nearly half a century. Underlying this preoccupation is the assumption that road traffic will naturally arrange itself in an equilibrium flow, under steady state conditions. In this context, predictions of future traffic flow patterns that would follow any changes to the network or demand levels rely upon an accurate representation of the traffic equilibrium.

Except for recent years, telecommunication network flow models have not made use of the notion of network equilibrium, as traffic networks do. Indeed, while road traffic is highly individualized, each driver making his or her own route choice decision, telecommunication networks are much more centralized.

Recently, however, equilibrium models have started to emerge in telecommunication networks. This is due to two main reasons. First, the deregulation and privatization of large telecom companies introduced competitive and decentralized behavior among telecom operators. Secondly, a new concept of networks has been developed and deployed with great success, in which most of the intelligence lies at the edges of the network (at the sources and destinations) allowing for increased speed and reduced overhead and costs, as well as the possibility for intelligent (and selfish) route or provider choice decisions. The Internet provides one example of such an environment.
1.1. Wardrop and Nash equilibria

The definition of the steady state equilibrium of a traffic network was put forth by J.G. Wardrop in his 1952 treatise (Wardrop, 1952) which provided two different definitions of traffic assignment concepts. The first is commonly referred to as the Wardrop, or traffic equilibrium, principle and, as we will show later, is a variant of Nash equilibrium for networks. It states that “The journey time on all the routes actually used are equal, and less than those which would be experienced by a single vehicle on any unused route.” Wardrop’s second principle is what has become known as the system optimum principle, and states that “The average journey time is a minimum.” While some point to the economist Pigou, who stated analogous principles in his 1920 *Economics of Welfare*, as the rightful originator of these ideas, they had not been applied to networks, and transportation networks in particular, until Wardrop’s seminal work.

Analogous to Wardrop’s first principle is the definition of the famous Nash equilibrium defined in J.F. Nash’s 1950 doctoral thesis. Expressing it in terms of network flows, a flow pattern is in Nash equilibrium if no individual decision maker on the network can change to a less costly strategy, or route. In other words, the Nash equilibrium does not state what can or cannot happen when more than one decision maker changes their strategy (route) simultaneously. When the decision makers in a Nash game are discrete and finite in number, a Nash equilibrium can be achieved without the costs of all used routes being equal, contrary to Wardrop’s equilibrium principle. In some cases, Wardrop’s principle represents a limiting case of the Nash equilibrium principle, as the number of users becomes very large. We will present some such precise statements on this relationship later in this paper.

The second Wardrop principle, that of system optimality, assumes that congested networks can be globally optimized. While this can be true for a network which is entirely controlled by a single operator, it is not so with networks of road traffic or with disaggregated telecommunication networks. Therefore, in the remainder of this paper, we will focus on equilibria, rather than pure system optima.

1.2. Equilibria in networks

Given the fundamental nature of equilibria in many large-scale systems, it is of no surprise that researchers studying transportation networks have been preoccupied with developing models that reproduce this equilibrium, as a function of network characteristics and user demand levels. Typically, transport equilibrium models consider vehicles to be the fundamental units seeking an equilibrium, or, in the case of public transport, the individual traveler. In both of these cases, since the number of users is generally very large, the Wardrop concept, that treats individual user contributions to the costs as infinitesimal, is preferred to the (in this respect, more general) Nash paradigm.

In the context of telecommunication networks, the Wardrop equilibrium is used most often to model the situation in which the routed entities are packets, and routing decisions are taken at the nodes of the networks (rather than by the users) so as to minimize the (per-packet) delay. In many actual networks, the routers at the nodes seek to minimize the per-packet delay in terms of the number of “hops,” or nodes, to the destination. There are, however, situations in which it is more advantageous to work with actual delays as