Analysis of a Communication Network Governed by an Adaptive Random Multiple Access Protocol in Critical Load

D. Yu. Kuznetsov* and A. A. Nazarov**

* Tomsk Polytechnic University
dima@ido.tpu.edu.ru
** Tomsk State University
nazarov@fpmk.tsu.ru

Received August 11, 2003; in final form, April 13, 2004

Abstract—A mathematical model of an adaptive random multiple access communication network is investigated. The value of the network critical load is found; in the critical load, asymptotic probability distributions for states of the information transmission channel and for the number of requests in the source of repeated calls are found. It is proved that distributions of the normalized number of requests belong to the class of normal and exponential distributions, and it is shown how conditional normal distributions pass in the limit to the class of exponential ones.

1. INTRODUCTION

To provide reliable data transmission in telecommunication networks, mathematical modeling is required with the purpose of finding optimum characteristics of transmitting facilities and designing algorithms guaranteeing message delivery in reasonable time. This problem is of most importance for random access networks, which operate rather irregularly in high load of communication channels.

For stabilizing such communication networks, various modifications of random access protocols are suggested [1–4]. Thus, in [1, 2], respectively, the asynchronous and synchronous models of a dynamic random multiple access network are investigated. Dynamic protocols give efficient results in stabilizing a communication network but are not realizable technically since each user must have information on the total number of retrial requests. In [4], a model of a synchronous random multiple access system with the part-and-try protocol is considered. Another approach to stabilization of random multiple access communication networks is realized by adaptive protocols [5–8]. Thus, [5] considers a model of a communication network similar to that described in the present paper; however, in [5] only one network characteristic, the capacity, is considered.

An efficient instrument of mathematical modeling for such communication networks is the queueing theory apparatus. With the help of it, analytical models of data transmission networks are constructed. Here, in contrast to classical queueing models, one has to consider retrial queueing systems [9]. Developing methods for analysis of retrial queues requires attention since such systems are at present widely used in modeling.

One of such methods is the asymptotic analysis method [10]. Using this method, in [6, 7] two models of adaptive random multiple access networks were analyzed: in overload [6]; when load $\rho$ of a communication channel is larger than some critical value $S$, for a network with a finite number, $N$, of users; and in large load [7], when $\rho$ tends to $S$ from below, with infinitely many users. Note that in the latter case $S$ has the sense of the capacity of the communication network.
In the present paper, we consider communication networks with \( N \) users in the asymptotic conditions \( N \to \infty \), when load \( \rho \) tends to \( S \) both from above and from below, which allows us to pass from the first model [6] to the second [7], and also to analyze cases with \( \rho = S \).

In the paper, asymptotic probability distributions of channel states and of the number of requests in the source of repeated calls (SRC) are found. It is proved that distributions of the normalized number of requests belong to the class of normal and exponential distributions, and it is shown how conditional normal distributions pass in the limit to the class of exponential ones.

2. MATHEMATICAL MODEL OF AN ADAPTIVE RANDOM ACCESS NETWORK

In [6], as a mathematical model of an adaptive random access communication network, it was suggested to consider a single-server queueing system fed by requests from \( N \) outer sources (users). Requests are generated by each source randomly; generating times are distributed exponentially with parameter \( \mu = \frac{N}{\rho} \). If, at the request arrival, the channel is free, the service starts; service time is random with distribution function \( B(s) \). If the channel is busy at the request arrival, both requests are distorted and enter the source of repeated calls; from this moment, distribution of the collision warning signal begins, whose duration has distribution function \( A(s) \). Requests that arrive during the warning signal distribution enter the SRC without distorting the warning regime in the channel. When the warning regime is completed, the channel becomes free again. The delay time in the SRC is random; its distribution has intensity \( 1/T \) (the sense of the parameter \( T \) is explained below).

After a random delay in the SRC, requests make repeat attempts to access the channel. We describe the state of the channel by a parameter \( k \), which assumes three values: \( k = 0 \) if the channel is free, \( k = 1 \) if it is busy with servicing a request (message transmission), and \( k = 2 \) if the channel is in the collision warning state. Denote by \( i \) the number of requests in the SRC.

The parameter \( T \), which determines the intensity of accessing the channel by requests from the SRC, coincides with the current state of the adapter which stabilizes the operation of the unstable random access network under consideration.

The process of changing the adapter state \( T(t) \) is defined as follows:

\[
T(t + \Delta t) = \begin{cases} 
T(t) - \alpha \Delta t & \text{if } k(t) = 0, \\
T(t) & \text{if } k(t) = 1, \\
T(t) + \beta \Delta t & \text{if } k(t) = 2.
\end{cases}
\]

Here, in contrast to [6], we consider a slightly different adapter construction (1).

We describe the state changing process of the adaptive communication network by the three-dimensional random process \( \{k(t), i(t), T(t)\} \).

In the present paper, we restrict ourselves to the analysis of a system where distribution functions \( B(s) \) and \( A(s) \) are exponential with the following parameters: \( \mu = 1 \) for the service time, and \( \mu_1 = 1/\nu \) for the collision warning signal duration.

In this case, the random process \( \{k(t), i(t), T(t)\} \) under consideration is a Markov process; that is why we call this mathematical model Markovian. The analysis method presented below can be generalized to the analysis of a non-Markovian model for arbitrary distribution functions \( B(s) \) and \( A(s) \) using the supplementary variable method [11].

For the analysis of the adaptive system constructed, denote

\[
P_k(i, t) = P_k(i, T, t) = P_k(i, T, t) dT.
\]

Similarly to [6], it is easily shown that the distribution of probabilities \( P_k(i, T, t) \) in the stationary regime,

\[
P_k(i, T) = P_k(i, T),
\]