CONTRIBUTIONS OF PSEUDOSCALAR PARTICLES TO THE ABNORMAL MOMENTS OF AN ELECTRON IN A PLANE WAVE FIELD

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In the present paper, the contributions of pseudoscalar particles to the abnormal magnetic and electrical moments of an electron moving in a plane wave field are considered.

In the present paper, the contributions of pseudoscalar particles (including the \(\pi^0\)-meson and the Higgs bosons) to the abnormal magnetic and electrical moments of a polarized electron moving in a plane electromagnetic wave field, representing a superposition of a constant crossed field \((E \perp H)\) and a plane electromagnetic elliptically polarized wave are considered. The vector potential of this field has the form

\[
A(\xi) = a\xi(e_1 \sin \beta + e_2 \cos \beta) - \frac{A_0}{\omega}(e_1 \cos \psi \sin \omega \xi - e_2 \sin \psi \cos \omega \xi).
\]

The angle \(\psi(-\pi \leq 2\psi \leq \pi)\) characterizes the wave polarization. Thus, \(2\psi = 0, \pi\) corresponds to a linearly polarized wave, and \(g = \pm 1\) characterizes clockwise (at \(g = 1\)) and counterclockwise wave polarizations (at \(g = -1\)); \(e_{\nu}(\nu = 1, 2)\) is the orthogonal reference vector in the plane orthogonal to the wave propagation vector \(\mathbf{n}: (e_1 e_2) = (ne_1) = (ne_2) = 0, n^2 = e_1^2 = e_2^2 = 1; \beta\) is the angle between the principal axes of the polarization ellipse and vectors \(\mathbf{E}\) and \(\mathbf{H}\) of the constant crossed field of strength \(a; \omega\) is the frequency; and, \(A_0\) is the wave amplitude.

The interaction of virtual electrons and pseudoscalar particles in transient states with external electromagnetic field (1) results in the radiation correction for the electron mass containing two terms caused by the abnormal magnetic moment (AMM) and the abnormal electric moment (AEM) of the electron. The constant crossed field in superposition (1) allows us to separate these two terms linearly dependent on the electron polarization \(\zeta\) and proportional to \((\zeta H)\) and \((\zeta E)\), which can be interpreted as the energy of interaction of the AMM with the constant magnetic field \(\mathbf{H}\) and of the AEM with the constant electric field \(\mathbf{E}\). Thus, the change in the electron mass \(\Delta m\) can be represented as \(\Delta m = \Delta m_1 + \Delta m_2\), where \(\Delta m_2\) comprises only terms linearly dependent on the electron polarization and proportional to \((\zeta H)\) and \((\zeta E)\). In a system at rest, we can write \(\Delta m_2 = -\mu_1(\zeta H) - \mu_2(\zeta E)\). Then the contributions of pseudoscalar particles to the abnormal moments (\(\mu_1\) to the AMM and \(\mu_2\) to the AEM) are determined by the expression

\[
\mu_k = \frac{\mu_0 G^2}{8\pi^2} \int_0^\infty \int_0^\infty \frac{t^2 dt}{(1 + t)^3} \sum_{S=-\infty}^{\infty} t^S J_S(x_1) J_S(x_2) e^{-i\eta} R_k \left[ \frac{1 + \nu}{2} \left( 1 + \frac{2}{t} \right) \delta_{\zeta,\zeta'} + \frac{\nu - 1}{2} \delta_{\zeta,-\zeta'} \right],
\]

where

\[
R_1 = \varphi - icb \sin xL_{2S}, \quad R_2 = \frac{b}{2c} \sin x \left( 2S x_2 \sin 2\psi - iL_{2S} \sin 2\beta \cos 2\psi \right), \quad x_1 = b^2 \frac{ht}{\lambda} \sin x \cos 2\psi.
\]
\[ \eta = \frac{t \varphi^3}{3 \chi} + \frac{\varphi}{\chi} \left[ 1 + \Delta^2 \frac{1 + t}{t^2} + \frac{1}{2} b^2 \left( 1 - \frac{\sin^2 x}{x^2} \right) \right] - 2s \alpha, \quad x_2 = 4be \frac{b t}{\chi} \varphi. \]

The above expressions are complex functions of all parameters characterizing superposition (1) and of the parameters of pseudoscalar particles and electron. The parameter \( \nu = \pm 1 \), and Eq. (2) at \( \nu = -1 \) determines the contributions of \( \pi^0 \)-mesons, whereas at \( \nu = 1 \), it determines the contributions of the Higgs bosons. The interaction constants are \( G_\pi = 2^{-1/2} m_\pi f_\pi G_F \cos \Theta \) and \( G_E = m_\pi / v \), respectively, where \( G_F \) is the Fermi constant, \( f_\pi \) is the dimensionality parameter of the mass [1], \( \Theta \) is the Cabibbo angle, and \( v = (\sqrt{2} G_F)^{-1/2} \) is the average Higgs vacuum field.

Terms proportional to \( \delta_{\zeta, \zeta'} \) in Eqs. (2) describe contributions of electron transitions to transient states without spin flip, and terms proportional to \( \delta_{\zeta, -\zeta} \) describe contributions of electron transitions with the electron spin flip.

Thus, the contributions to the abnormal moments caused by the virtual emission and absorption of \( \pi^0 \)-mesons come from the transitions to the transient states only with the electron spin flip \( (\nu = -1) \), and the contributions from the Higgs bosons come only from the transitions without electron spin flip \( (\nu = 1) \).

The sum over \( S \) in Eq. (2) is removed if a circularly polarized wave is presented in superposition (1), because \( x_1 = 0 \) at \( 4 \psi = \pi g \), and the Bessel function \( J_0 (0) = \delta_{S,0} \), where \( \delta_{m,n} \) is Kronecker’s delta symbol. For \( \beta = 0 \), the sum over \( S \) in Eq. (2) is also removed for a linearly polarized wave, because \( \psi = 0 \) and \( c = 0 \) at \( \beta = 0 \); then \( x_2 = 0 \) and \( J_{2S} (0) = \delta_{S,0} \). The contribution to the AEM vanishes at \( \beta = 0 \), since in this case the value \( R_2 \) is proportional to \( S \) and vanishes after summation over \( S \). If the plane wave in Eq. (2) is circularly polarized, the contribution to the AEM also vanishes, since in this case, \( S = 0 \), \( L_{2S} = 0 \), and \( R_2 = 0 \).

In Eqs. (2), we have introduced the following designations:

\[
\chi = \frac{e a}{m} (nq), \quad b = \frac{e a_0}{m \omega}, \quad \lambda = \frac{2 \omega}{m^2} (nq), \quad h = \cos x - \frac{1}{x} \sin x, \quad x = \frac{\lambda \varphi}{2 \chi},
\]

\[
L_m = \frac{J'_m}{J_m}, \quad \tan \alpha = \tan \beta \cot \psi, \quad 2 \varphi^2 = 1 - \cos 2 \psi, 2 \beta, \quad \mu_0 = \frac{e}{2m}.
\]

The abnormal electric moment of the electron induced by superposition (2) was studied in [2, 3], the AEM of the electron induced by an arbitrary constant electromagnetic field \((E H) \neq 0\) was studied in [4], and the AEM induced by the neutrino was studied in [5, 6].

Let us consider now some particular cases for limiting values of external field (1). For a weak plane wave intensity \( b \ll 1 \) and a low strength of the constant crossed field \( \chi \ll 1 \), expanding Eq. (2) in a series, we obtain

\[
\mu_1 = \mu_1^0 + \mu_1^a + \mu_1^b, \quad \mu_2 = \mu_2^b,
\]

\[
\mu_1^0 = \frac{\mu_0 G^2}{32 \pi^2} [\delta_{\zeta, \zeta'}(1 + \nu) I_1 + \delta_{\zeta, -\zeta'}(\nu - 1) I_2],
\]

where

\[
I_1 = 3 - 2 \Delta^2 + \Delta^2 (\Delta^2 - 3) \ln \Delta^2 + \frac{\Delta^2 - 1}{\Delta^2 - 3} \ln \frac{1 - \tau}{1 + \tau}, \quad \tau = \sqrt{1 - 4 \Delta^2},
\]

\[
I_2 = 1 + 2 \Delta^2 + \Delta^2 (1 - \Delta^2) \ln \Delta^2 - \frac{\Delta^2}{\tau} (\Delta^2 - 3) \ln \frac{1 - \tau}{1 + \tau}.
\]

In the limiting case of \( \Delta \gg 1 \), we have

\[
I_1 = \frac{2}{\Delta^2} \left( \ln \Delta^2 - \frac{7}{6} \right), \quad I_2 = \frac{2}{\Delta^2} \left( \ln \Delta^2 - \frac{11}{6} \right).
\]

For the parameter \( \mu_1^a \) we then obtain:

\[
\mu_1^a = \mu_0 \frac{\chi G^2}{8 \pi^2 \Delta^2} \left[ \delta_{\zeta, \zeta'}(1 + \nu) \left( \frac{1}{3} - \frac{5}{\Delta^2} \ln \Delta^2 + \frac{197}{12 \Delta^2} \right) + \delta_{\zeta, -\zeta'}(\nu - 1) \left( \frac{1}{3} - \frac{7}{\Delta^2} \ln \Delta^2 + \frac{1499}{60 \Delta^2} \right) \right].
\]

Here we have considered only the terms comprising \( \Delta^{-2} \).