QUASI-RAYLEIGH WAVES IN TRANSVERSELY ISOTROPIC HALF-SPACE WITH INCLINED AXIS OF SYMMETRY

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ABSTRACT

A method for determination of characteristics of quasi-Rayleigh (qR) wave in a transversely isotropic homogeneous half-space with inclined axis of symmetry is outlined. The solution is obtained as a superposition of qP, qSV and qSH waves, and surface wave velocity is determined from the boundary conditions at the free surface and at infinity, as in case of Rayleigh wave in an isotropic half-space. Though the theory is simple enough, a numerical procedure for calculation of surface wave velocity presents some difficulties. The difficulty is caused by necessity to calculate complex roots of a non-linear equation, which in turn contains functions determined as roots of non-linear equations with complex coefficients. Numerical analysis shows that roots of the equation corresponding to the boundary conditions do not exist in the whole domain of azimuths and inclinations of the symmetry axis. The domain of existence of qR wave depends on the ratio of the elastic parameters: for some strongly anisotropic models the wave cannot exist at all. For some angles of inclination qR-wave velocities deviate from those calculated on the basis of the perturbation method valid for weak anisotropy, though they have the same tendency of variation with azimuth. The phase of qR wave varies with depth unlike Rayleigh wave in an isotropic half-space. Unlike Rayleigh wave in an isotropic half-space, qR wave has three components - vertical, radial and transverse. Particle motion in horizontal plane is elliptic. Direction of the major axis of the ellipse coincides with the direction of propagation only in azimuths 0° (180°) and 90° (270°).

Keywords: transverse isotropy, Rayleigh waves, velocity, polarization

1. INTRODUCTION

Different seismological observations indicate that most parts of the Earth are anisotropic. The effect of anisotropy on seismic surface waves is manifested in two phenomena: (1) Rayleigh-Love wave discrepancy - impossibility to explain Rayleigh and Love wave dispersion by the same isotropic model (Anderson, 1961; McEvilly, 1964); (2) azimuthal anisotropy derived from azimuthal variation of phase and group velocities (Forsyth, 1975; Tanimoto and Anderson, 1985; Suettsugu and Nakanishi, 1987; Nishimura and Forsyth, 1989; etc.) as well as from anomalous polarization of surface waves (Yu and
The first kind of the phenomena is explained in the framework of a transversely isotropic model with vertical axis of symmetry, usually referred to as radial anisotropy. The second one (azimuthal anisotropy) is considered as a result of the most general case of anisotropy, but due to mathematical and computational difficulties it is treated approximately, only for weak anisotropy (Smith and Dahlen, 1973, 1975; Maupin, 1989) by the perturbation technique. This technique allows to express a variation of phase velocity with azimuth $\phi$ as a composition of trigonometric functions of $2\phi$ and $4\phi$. Such representation of phase velocity is commonly used in different seismological studies (Tanimoto and Anderson, 1985; Montagner and Nataf, 1986; Montagner and Tanimoto, 1990; Park, 1996).

Crampin (1970) extended the Tomson-Haskell matrix method for a multilayered half-space with anisotropic layers of general type. However the procedure for construction of the dispersion equation was described in a general form, and due to computational difficulties it was impossible to conclude on a behavior of the surface wave field in any specific structure. Farnell (1970) has presented numerical solutions for a homogeneous anisotropic half-space with one of crystal axes directed perpendicular to the free surface. The existence of Rayleigh wave in such a model with velocity less than minimum of three body wave velocities was proved by Lothe and Barnett (1976). However, these results cannot be applied to a case, when one of the axes (symmetry axis for transversely isotropic medium) is not perpendicular to the free surface.

In the present paper we use the approach similar to that proposed by Crampin (1970) for a simple type of anisotropy - transversely isotropic medium with inclined axis of symmetry and for the simplest model of a homogeneous half-space. Analytical and numerical analysis allows us to display some properties of the wave field, and to check the accuracy of the approximate solution for weak anisotropy.

2. QUASI-RAYLEIGH WAVE IN A HOMOGENEOUS ANISOTROPIC HALF-SPACE

2.1. Formulation of the problem

We consider a homogeneous half-space $z>0$. The material of the half-space is transversely isotropic, but the symmetry axis $z'$ does not coincide with vertical axis $z$: the angle between $z$ and $z'$ is $\theta$ (Fig. 1a). Hereafter we use the conventional notation for the elastic parameters $A, C, F, N, L$. We shall consider two Cartesian coordinate systems: $xyz$ related to the half-space, and $x'y'z'$ with $z'$ along the symmetry axis. The axes $x$ and $x'$ are placed in the $zz'$ plane, the axes $y$ and $y'$ coincide and are orthogonal to the plane $zz'$.

The stress-strain relation in the system $xyz$ may be written in the form

$$\tau_i = C_{ij} \varepsilon_j,$$

where the indices $i$ and $j$ vary from 1 to 6 in accordance with the following rule: $1 \rightarrow (11)$, $2 \rightarrow (22)$, $3 \rightarrow (33)$, $4 \rightarrow (23)$, $5 \rightarrow (13)$, $6 \rightarrow (12)$, i.e. $\varepsilon_1 = \varepsilon_{11}$, $\varepsilon_2 = \varepsilon_{22}$, ..., $\varepsilon_6 = 2\varepsilon_{12}$.

Expressions for elements of the matrix $C$ are given in Appendix 1.