In one fell swoop, Kenneth Arrow helped answer a number of unresolved or only partially resolved questions. In parallel with the work of Gerard Debreu, he provided the first truly general competitive equilibrium existence proof. He also incorporated an extension of the von Neumann-Morgenstern expected utility hypothesis into a model that has since become the standard of analysis in finance. Finally, he provided the first formal model of futures markets and he explicitly described the ways in which uncertainty and information relate in general equilibrium.

The Nobel Prize

After Arrow was told that he would win the Nobel Prize, his wife shared with the wife of Franco Modigliani that one must, upon receiving the famous early morning phone call from the Committee in Stockholm, immediately shower and shave, as the media would be at the doorstep within minutes.155 Such was the experience of Arrow, his wife Selma, and their children David and Andrew in the fall of 1972. For his contributions, he shared the 1972 Nobel Prize with John Hicks. The Sveriges Riksbank Prize Committee in Economic Sciences in honor of Alfred Nobel made the award for their “pioneering contributions to general economic equilibrium theory and welfare theory.”156 In his Nobel address, Arrow spoke of the contributions of others before him, including John von Neumann, in providing the tools of general equilibrium analysis. He also described his own contributions, documented here and recognized by the Nobel Prize Committee, and outlined work that remained to do.

While the Prize is typically given to one whose life’s work makes a significant and lasting contribution in the decision-making sciences,
Arrow’s most remembered work flowed from an intense period of insight while he prepared his PhD thesis at Columbia University. In essence, he had secured his Nobel Prize before he had even established himself at his adopted home at Stanford University.

The environment for his insights was the Cowles Commission for Research in Economics at the University of Chicago. This was the same hotbed of intellectual activity directed by Jacob Marschak and Tjalling Koopmans that would also result in Nobel Prizes for Koopmans, Milton Friedman, Leonard Jimmie Savage, Gerard Debreu, Lawrence Klein, and the young Harry Markowitz, all of whom were at Cowles or Chicago in the late 1940s and early 1950s.

Meanwhile, as this group grappled with general equilibrium analysis, the Princeton Institute for Advanced Study and Princeton University were advancing the study of game theory and linear programming. Together, scholars at these two locations defined and forever changed finance in just half-a-dozen years.

Closely related to Arrow’s work on general equilibrium theory was Arrow’s Impossibility Theorem, which was the subject of his PhD thesis. In an attempt to determine whether the problem of the determination of general equilibrium could be simplified through the characterization of a representative social utility function, Arrow discovered that such a welfare function is logically inconsistent.

An impossibility theorem

Arrow’s Impossibility Theorem was motivated by the need to discover a preference ordering that was representative of a group larger than the individual and over three or more alternatives. Each of these individuals brought to the preference ordering their convex and consistent utility functions over the choices at hand. They would then vote on the preferred outcome for which a social welfare function could mimic. In his axiomatic approach, Arrow imposed a few reasonable conditions for a social welfare function candidate:

- **Non-dictatorship**
  The social welfare function cannot effectively impose the preferences of a single individual on to the entire group.

- **Monotonicity**
  A change of any individual’s ranking in favor of one decision should never result in the overall ranking diverging away from preferring that decision.