In this chapter we use "The Human Income-Expenditure Balance in a Non-Monetary (Robinson Crusoe) Economy" (Figure 1.6 of Chapter 1, Section 1) to construct a theoretical framework in which output and the allocation of resources are simultaneously determined as a result of the process whereby a person maximizes expected total utility (the sum of expected utility from goods and utility of leisure) against a backdrop of diminishing utility, and derive the "basic equation of labor" for a non-monetary economy.

Assumptions Behind Our Analysis

The analysis in this chapter is based on the following simplifying assumptions:

(1) In cases where Crusoe consumes a number of consumption goods on an ongoing basis, the utility associated with a given consumption good depends solely on the amount of that good consumed by Crusoe, i.e. it does not depend on the amounts of other consumption goods consumed by Crusoe.

(2) Expected utility $E_1$ (a measure of the satisfaction derived by Crusoe from consumption of a given good) is not affected by time spent resting ($T_R$), nor is utility from leisure $E_2$ (a measure of the satisfaction derived by Crusoe from time taken to restore productive capacity) affected by time spent working ($T_L$). In other words, expected utility $E_1$ depends solely on time spent working ($T_L$), and utility from leisure $E_2$ depends solely on time spent resting ($T_R$).

(3) As consumer, Crusoe consumes two different consumption goods (Good 1 and Good 2), while as producer, Crusoe produces two different consumption goods (Good 1 and Good 2) and one pure investment good (Good 3).

(4) Crusoe's utility attrition function for each consumption good is linear (that is, each consumption good has a constant utility attrition rate).
(5) A capital stock exists at the start of the analysis period, and the capital attrition rate is zero. In terms of Figure 1.6, this means that the proportion allocated to capital is zero, and there is no flow of "replacement investment". In other words, all output is allocated to Crusoe.

(6) Pure investment during the analysis period does not add to the capital stock, and has no impact on utility attrition rates or labor productivity during the analysis period.

Assumptions (1) and (2) ensure additivity of utility, such that in cases where Crusoe consumes a number of consumption goods on an ongoing basis, expected utility from these goods $E_1$ can be calculated as the sum of the utilities associated with each individual consumption good, while expected total utility $E$ (a measure of the overall satisfaction felt by the consumer) can be expressed as the sum of expected utility from goods ($E_1$) and utility from leisure ($E_2$). In other words:

$$E_1 = AU_1(q_1^C) + AU_2(q_2^C) + \cdots + AU_n(q_n^C)$$

$$E = E_1(T_L) + E_2(T_R)$$

### Assumptions Relating to Production

Our analysis assumes a Cobb-Douglas production function of form

$$q_i = b_i T^{1-a}_\mu$$

where the production elements are the capital stock and labor time. For $\alpha > 0$ (i.e. $1 - \alpha < 1$) this production function satisfies the principle of diminishing marginal productivity.²

The shape of this production function depends on the specific value of $\alpha$. The short-term equilibrium analysis in this chapter assumes that $\alpha = 1/2$, such that the production function takes the form $q_i = b_i T^{1/2}_\mu$.

### Crusoe's Behavioral Principles

Crusoe’s behavior as a consumer and a producer is governed by the following principles.

1. For a given “allocation to labor” (out of the total output of goods supplied by Crusoe as producer) and a given proportion of this allocation that is apportioned to consumption (“the average propensity to consume”, denoted by $a$), Crusoe as consumer allocates his time ($T$) across time spent working ($T_L$) and time spent resting ($T_R$) so as to maximize his total expected utility (the sum of utility from goods and utility of leisure). Time equivalent to the amount required