‘Well Maybe Not Exactly, but It’s Around Fifty Basically?’: Vague Language in Mathematics Classrooms

Tim Rowland

It may come as something of a surprise to find a mathematician (albeit in the guise of a mathematics educator) writing about vagueness, since it is commonly supposed that precision is the hallmark of mathematics. Such a point of view is reflected in the landmark 1982 Report of the Committee of Inquiry into the Teaching of Mathematics in Schools (the Cockcroft Report), which asserted that, ‘mathematics provides a means of communication which is powerful, concise and unambiguous’ (Department of Education and Science 1982, p. 1), and proposed the communicative power of mathematics as a ‘principal reason’ for teaching it. There was refreshing novelty in such a claim, which seemed to be justifying the place of mathematics in the curriculum in much the same way that one might justify the learning of a foreign language, and it did much to promote and sustain interest in the place of language in the teaching and learning of mathematics. Such a view of mathematics is in contrast, however, with that expressed in a contemporary pamphlet issued by the Association of Teachers of Mathematics (ATM 1980, pp. 17–18), whose authors argued that:

Because it is a tolerant medium, everyday language is necessarily ambiguous. / . . ./ Now, mathematising is also a form of action in the world. And its expressions, however carefully defined, have to retain a fundamental tolerance / . . ./ Because it is a tolerant medium, mathematics is also necessarily an ambiguous one.

This description emphasizes mathematics as human activity, and language as a social means of working towards mutual understanding and agreement. Furthermore, it offers the radical proposal that ambiguity is a beneficial ingredient in the formulation, the ‘expression’, of
mathematics. As a ‘product’ (polished, final), mathematics may be pre­sented, particularly in writing but also in speech, as though it lacked ambiguity, representing truths about the world – or at the very least, about itself – in a sure, exact and unequivocal kind of way. By contrast, the ‘process’ of mathematics production (‘mathematizing’) is character­ized by a number of forms of vagueness.

Consider the following brief episode from a mathematics lesson in a secondary school. The teacher, Judith, is working with a class of 13- to 14-year-old pupils, who have marked a number of dots (points) on paper. The pupils’ task is to connect them, by drawing line segments joining two points. They are asked, ‘What is the smallest number of segments necessary to join all of the points?’ It is rather like a cluster of towns and a network of roads that joins them all. One pupil, Allan, starts with a $3 \times 3$ array of dots, and counts eight line segments. Judith enquires:

Judith: All right then, so what’re you going to do now?
Allan: I’ll try a, um, 4 by 4 grid.
Judith: Right. Can you make any predictions before you start? (3 seconds’ pause)
Allan: The maximum will probably be, er, the least’ll probably be about 15.
Judith: So why did you predict 15?
Allan: Uh . . . because I thought there might be a pattern between . . . if there was um a certain amount of, um . . . if it’s 3 by 3 say . . .
Judith: Uh-hum.
Allan: If you ti-, 3 times 3 is actually 9.
Judith: Uh-hum.
Allan: But as, if you went round all the dots, it would only come to about, if you did it once it would come to one, uh less than 9, ‘n’ you got, uh, because, because there’s o-, there’s only . . . ‘cause you only have, y- . . . you can miss out a line exactly, ‘cause you, you can miss out a gap, c- ’cause you um, y’d ’ave to go all the way round the whole dots.
Judith: OK . . . So why did that make you say 15?
Allan: Because uh, f- for the same reason, ’cause if you um w- tried to go round the whole all the dots you’d get 16 but if you just did it once all the way round the dots but missing out gaps you’d still come to uh, you just minus one basically and just . . .
Judith: So what would happen in some other squares?