2
Some Important Spatial Definitions

2.1 The Spatial Weight Matrix $W$ and the definition of Spatial Lag

If some autocorrelation is present in the stochastic disturbances, some or all the off-diagonal elements of the variance-covariance matrix are non-zero. In such a situation, as stated earlier, the optimal properties of the OLS are not valid and the GLS criterion can only be applied if we are able to specify a plausible form of autocorrelation. In view of this, the following chapters will consider various alternatives to model non-diagonal variance-covariance matrices when data are observed in geographical units such as countries or regions. In this section we will introduce some preliminary concepts.

When we observe a phenomenon in say, $i=1,\ldots,n$ regions, non-diagonal variance-covariance matrices arise from the presence of spatial autocorrelation among the stochastic terms. Positive spatial autocorrelation arises when units that are close to one another are more similar than units that are far apart. Similarly, $VC$ matrices can also display spatial heterogeneity when some areas present more variability than others. As an example see Figure 2.1.

In the definition of spatial autocorrelation we mentioned the concept of closeness which requires some further specification. Indeed, the major difference between standard econometrics and spatial econometrics lies in the fact that, in order to treat spatial data, we need to use two different sets of information.

The first set of information relates to the observed values of the economic variables whereas the second set of information relates to the particular location where those variables are observed and to the various links of proximity between all spatial observations. The presence of
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This extra set of information related to space is also the reason why the standard econometric and statistical packages (for example, Eviews or SPSS) are so reluctant to introduce dedicated modules to spatial econometrics and spatial statistics which require extra capabilities in order to deal with spatial maps. If data are observed on a regular square lattice grid, like the one shown in Figure 2.1, closeness can be straightforwardly defined by choosing between the so-called rook criterion (two units are close to one another if they share a side) or the queen criterion (two units are close to one another if they share a side or an edge), drawing on the chess move analogues illustrated in Figure 2.2.

Figure 2.1 Spatial autocorrelation and spatial heterogeneity among 64 spatial units arranged in an 8-by-8 regular square lattice grid. Different greytones refer to different values of the variables under study ranging from low values (white) to high values (black). (a) Spatial autocorrelation. (b) Spatial heterogeneity. Left pane: high variability. Right pane: low variability

Figure 2.2 Contiguity criteria in a regular square lattice grid. (a) Rook’s move and (b) Queen’s move