Forward Implied Volatilities

For path-dependent and forward starting options, it is important to assess Vega, the sensitivity of the option's value to changes in volatility, and in particular to assess these sensitivities for forward buckets. A first step in this process is to determine how forward volatilities for these forward buckets are calculated from the spot volatilities implied in current market option prices. The procedure for determining these forward implied volatilities is similar to the procedure used for determining forward rates from spot prices. Given the spot rates for a zero coupon security maturing at time 1 and a zero coupon security maturing at time 2, (where time 1 < time 2), it is possible, through bootstrapping, to calculate the forward rate for the period between time 1 and time 2. Similarly given the spot implied volatilities for the periods $t_0$ to $t_1$ ($\sigma_{t_0, t_1}$) and $t_0$ to $t_2$ ($\sigma_{t_0, t_2}$) respectively, it is possible to infer the expected volatility between $t_1$ and $t_2$ ($\sigma_{t_1, t_2}$). This volatility is the forward implied volatility (also known as the forward-forward volatility) for the period $[t_1, t_2]$.

In *Dynamic Hedging*, Taleb presents the formula\(^1\) for computing the annualized forward implied volatility for the period between $[t_{n-\alpha}, t_n]$, that is, $\sigma_{t_{n-\alpha}, t_n}$, as follows:

$$
\sigma_{t_{n-\alpha}, t_n} = \sqrt{\frac{\sigma_{t_0, t_n}^2(t_n - t_0) - \sigma_{t_0, t_{n-\alpha}}^2(t_{n-\alpha} - t_0)}{(t_n - t_{n-\alpha})}}
$$

where

- $\sigma_{t_0, t_n}$ is the annualized spot implied volatility for the period $t_0$ to $t_n$ for options expiring at time $t_n$
- $\sigma_{t_0, t_{n-\alpha}}$ is the annualized spot implied volatility for the period $t_0$ to $t_{n-\alpha}$ for options expiring at time $t_{n-\alpha}$

This formula accounts for unequal non-overlapping time steps, in line with how spot implied volatilities and options prices are quoted in the market.

For example, let us consider the following annualized spot implied volatilities for at the money call options of strike US$272 on Barclays stock (BARC) for 31 January 2014 obtained from ivolatility.com.
where \( t_0 \) is time 0.

Using the data provided, first calculate the annualized variance for the periods \((t_0, t_n)\), that is, \(\sigma^2_{t_0, t_n}\).

Then calculate the annualized forward variance for the period \([t_n, t_n - \alpha]\), \(\sigma^2_{t_n, t_n - \alpha}\).

For the first period, 0–30, the forward variance will be the same as the spot variance. For later periods, 30–60, 60–90, 90–120, … 360–720 and 720–1080, the forward variance will be calculated using Taleb’s formula, as follows:

\[
\sigma^2_{t_n, t_n - \alpha} = \frac{\sigma^2_{t_0, t_n}(t_n - t_0) - \sigma^2_{t_0, t_n - \alpha}(t_n - \alpha - t_0)}{(t_n - t_n - \alpha)}
\]