Exposure Measurement for Collateralised Portfolios

So far we have discussed how to model the exposure of a portfolio without any collateral, but we have seen that most of the netting sets these days are collateralised. In this section we will discuss how to account for collateral when calculating exposure metrics.

To summarise some of the details previously explained about collateral, we have seen that the legalities of a collateralised facility between two counterparties are typically set in a CSA agreement, that we can have one-way or two-way CSA agreements, and that collateral can consist of cash (in any currency that the CSA defines) or other securities like bonds or equities. Also, we explained that even in a perfectly collateralised facility we are exposed to a “gap risk”, which represents the potential change in value of the netting set (or of the hedges) from the default time until the book is liquidated.

4.1 Simulation steps

In order to take account of the collateral in the simulation, we need to follow the subsequent steps:

1. Simulate collateral per scenario and time step
2. Calculate the gap risk per scenario and time step
3. Calculate the exposure risk metrics

4.1.1 Simulating collateral

First of all, we need to simulate the collateral posted or received under the CSA. To achieve this we need to consider all the CSA mechanics and replicate them as accurately as possible at each scenario. The key elements to consider are one-way vs. two-way CSA, margining frequency, Threshold ($Thr^\ominus$), Initial Margin ($IM$), Minimum Transfer Amount ($MTA$), and Rounding.

At each time point in the Monte Carlo simulation we are going to look at the value of the netting set, as calculated by the Monte Carlo simulation, and we are going to simulate collateral posted and received forward in time so that we end up with a grid of collateral values.

CSA mechanics: For example, let’s say that the netting set consists of a portfolio of swaps and that the CSA has the following characteristics: bilateral, daily margining, our threshold of $5m$ ($Thr^\ominus = -5,000,000$), counterparty threshold of $1m$ ($Thr^\oplus = 1,000,000$), $^1$MTA of $100k$, and rounding of $10$.

Let’s say that the netting set starts with a value of zero ($V_{NS,0} = 0$). After one day, the value of the netting set goes to $-2,478,397$, which is above the $-5,000,000$, so no collateral call is made. After one day the value of the netting set goes to $-5,523,538$, which is below our threshold $Thr^\ominus$, so we need to post $523,540$ as
collateral, taking rounding into account. One day later the value of the netting set goes to −$836,912, within
the threshold levels, so we are returned the $523,540 we had posted. One day later $V_{NS}$ goes to +$598,509;
it is again within the threshold levels, so no collateral is transferred. The next day $V_{NS}$ goes to $1,054,209,
which is above the threshold $Thr^\oplus$, but the collateral required, $54k$, is less than the MTA, so no collateral is
transferred. Then $V_{NS}$ goes to $2,367,324$, above $Thr^\oplus + MTA$, so we call for $1,367,320$ of collateral— and we
continue like this at each scenario, time step by time step, until all trades have matured.

**Collateral algorithm:** Basically, the algorithm needs to work as follows in each scenario, iterating through the
time steps.

To start with, let’s say that we can simulate on every margin call day. Let’s refer to the current time step $t$ with
the index $i$, the value of the collateral at $t_i$ before a collateral call has been made as $\tilde{C}^\text{before}_i$, and the
value of the collateral at $t_i$ after a collateral call has been made as $\tilde{C}^\text{after}_i$. We are going to denote as $\tilde{C}$ the value
in the currency of the collateral, while $C$ is the value in the base currency of the financial institution. The
simulations with the prices of the netting set in each scenario and time point $t_i$ ($V_{NS,i}$) are already in the base
currency of the financial institution.

1. First we need to calculate the value of the collateral held in the CSA facility before any collateral call has
been made. This is given by

$$\tilde{C}^\text{before}_i = \Xi_{i-1 \rightarrow i}(\tilde{C}^\text{after}_{i-1})$$

where $\Xi_{a \rightarrow b}(\cdot)$ refers to the “time evolution” operator from time point $a$ to time point $b$. It simulates the
change in value of the collateral over time. This operator is the Unity operator (i.e., does not do anything)
when the collateral is cash. The operator does not account for changes of FX rates; that is covered in the
next point.

2. Then we need to transfer the value of the collateral to the base currency used in the simulation, multiplying
it by the relevant FX rate at time point $i$,

$$C^\text{before}_i = \tilde{C}^\text{before}_i \cdot FX_i$$

3. Now we need to check if there is going to be any collateral call. If

$$V_{NS,i} > \text{Threshold}_{\text{counterparty}} \rightarrow \text{We call collateral}$$

or if

$$\Delta C_i = V_{NS,i} - C^\text{before}_i - \text{Threshold}_{\text{counterparty}}$$

or if

$$V_{NS,i} > \text{Threshold}_{\text{ours}} \rightarrow \text{We post collateral}$$

$$\Delta C_i = V_{NS,i} - C^\text{before}_i - \text{Threshold}_{\text{ours}}$$

where $\text{Threshold}_{\text{counterparty}}$ is a number greater than or equal to zero and $\text{Threshold}_{\text{ours}}$ is smaller than or
equal to zero.