REFERENCES


A GRAPHICAL APPROACH TO PRICE BREAK ANALYSIS

In any stock control situation the main problem to be resolved is how many units of an item should be purchased at any one time, the “Economic Batch Quantity”, and having determined the EBQ, the problem of quantity discounts often arises. Would it be profitable to order in quantities other (usually more) than the EBQ in order to take advantage of a price discount?

The annual total cost (excluding stock-out cost) is given by:

\[ T = \frac{DS}{Q} + \frac{iCQ}{2} + DC, \]  

(1)

where \( Q \) is the batch size; \( D \), the rate of usage (in units p.a.); \( S \), the purchasing cost (per order); \( i \), the holding cost as a proportion of unit cost (p.a.); and \( C \), the unit cost. From which is derived the EBQ:

\[ Q_0 = \sqrt{\frac{2DS}{iC_0}} \]  

(2)

at a total cost given by:

\[ T_0 = \frac{DS}{Q_0} + \frac{iC_0Q_0}{2} + DC_0, \]  

(3)

where \( C_0 \) is the unit cost at the EBQ.

Let \( Q_1 \) be a proposed order quantity at a price \( C_1 \) and define:

\[ Q_1 = xQ_0, \]  

\[ C_1 = yC_0, \]  

(4)

where, in most cases, \( x > 1 \) and \( 0 < y < 1 \).

For the new order quantity \( Q_1 \), the annual total cost is given by:

\[ T_1 = \frac{DS}{Q_1} + \frac{iC_1Q_1}{2} + DC_1, \]  

(5)

and the change in total cost, equations (3)–(5), is:

\[ T_0 - T_1 = DS \left( \frac{1}{Q_0} - \frac{1}{Q_1} \right) + \frac{i}{2} (Q_0C_0 - Q_1C_1) + DC_0(1 - y). \]

This gives:

\[ T_0 - T_1 = DC_0 \left[ 1 - \frac{1}{x} \right] (1 - xy) + (1 - y), \]

(6)

\[ 300 \]
where

\[ k = \sqrt{\frac{2DC_0}{iS}}. \]  \hspace{1cm} (7)

For indifference, \( T_0 - T_1 = 0 \) and it may be shown that (6) reduces to:

\[ y = \frac{x(k+2)-1}{x(x+k)}. \]  \hspace{1cm} (8)

A graph of equation (8) with \( k = 1 \) takes the form shown in Figure 1. A particular discount situation may be represented by a pair of co-ordinates \((x, y)\) on the graph. Usually one is concerned with the region given by:

\[ 1 < x, \quad 0 < y < 1 \]

and this region is split by the curve such that any point falling above the curve represents an unprofitable discount and any point falling below a profitable one. As stated above, any point falling on the curve represents an indifference situation of no change in total cost.

The shape of the curve depends upon the parameter \( k \) given by equation (7) which may be written as:

\[ k = \sqrt{\frac{2V}{iS}}, \]  \hspace{1cm} (9)

where \( V = DC_0 \) is the value of the annual usage of the item. If one regards \( i \) and \( S \) (the stockholding and ordering costs) as fixed for a range of stock items, it is possible to plot a family of curves for that range corresponding to different values of \( V \), the annual usage. Therefore, when assessing the profitability of a particular discount it is only necessary to compare the point produced with the appropriate curve of the family which is determined by the annual usage of the item in question.

In certain situations, the problem is not merely one of assessing the profitability of a single discount but of deciding which (if any) of several possible