An infinite horizon mathematical programming model of a multicohort single species fishery

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Fishery policy evaluation should take account of the initial state of the fishery and the population dynamics of the fish stock. Although multicohort bioeconomic fishery policy evaluation models have been developed, the results from these models depend on the choice of planning period and the desired state of the stock at the end of this period. In this paper it is noted that these limitations can be overcome by evaluating fishery policy over an infinite time horizon, and a mixed integer programming (MIP) model is developed for carrying out this form of analysis in a multicohort single species fishery. This new MIP model allows policies to be evaluated over an infinite horizon by incorporating results from a steady state fishery model into a multiperiod framework. The use of this MIP model in determining policies for reaching and maintaining a steady state is illustrated.

Keywords: fisheries management; infinite horizon model; mathematical programming

Introduction

Fisheries management has increased in importance as more marine fisheries come under pressure from overfishing, and this has stimulated the development of model based methods for fishery policy evaluation. The simplest of these approaches are based on the Schaefer fish stock model (see Clark\(^1\)), but since the stock is represented in terms of total biomass, namely the weight of the stock, the age dependent development of each age class, or cohort, is not considered. This model has, however, been widely used for the analysis of sustainable yield, namely the fish catch that can be maintained in the long run from a fishery in a steady state, but the failure to consider policies for reaching a steady state is a fundamental weakness of all forms of sustainable yield analysis. Fishery policy evaluation should take account of the initial state of the fishery and the population dynamics of the stock, and should therefore be based on a multicohort fishery model.

The most widely used multicohort fishery model is the Beverton–Holt population model (see Clark\(^1\)) which is based on the assumption that the instantaneous natural and fishing mortality rates of each cohort are constant throughout any time period. This assumption is not valid over the annual time period generally adopted, but this multicohort model has been used in a number of deterministic multiperiod bioeconomic fishery models based on simulation\(^2,3\) and, in spite of computational difficulties arising from the form of this population model, optimisation methods\(^4\text{-}7\). Stochastic models are not widely used because of limited data availability. Helgason and Ólafsson\(^2\) argue that simulation is more appropriate than optimisation as the basis of a tool for fisheries management because the problems are multifaceted and optimisation results are perceived as being directive. However, if all relevant factors are incorporated in an optimisation model and the results are presented in an appropriate way, optimisation offers advantages. In particular, it is not necessary to run an optimisation model many times to evaluate policy. Simulation may, however, be useful in helping decision makers improve their understanding of policy options\(^8\).

The limitations of the Beverton–Holt population model were addressed by Glen\(^9\) who used a population model in which the annual natural and fishing mortality rates are expressed as proportions of the stock at the start of a year to develop a multiperiod mixed integer programming (MIP) model of a single species fishery. However, the planning period and the minimum spawning stock biomass at the end of this period must be specified in this MIP model, although both these factors influence the solution. In this paper it is shown that these weaknesses can be overcome by evaluating fishery policy over an infinite horizon. A new MIP model for this form of analysis is developed by incorporating results from the steady state fishery model developed by Glen\(^10\) into a multiperiod framework. This new infinite horizon model allows policies for reaching and maintaining a steady state to be determined, and the use of this model is illustrated.

Single species fishery models

Consider a multicohort single species fishery with \(n_i\) fish of age class \(i\), \(i = 0, 1, \ldots\), at the start of year \(t\), \(t = 1, 2, \ldots\),
recruitment \( r_i \) from spawning to age class 0 in year \( t \), and \( x_{it} \), fish of age class \( i \) caught in year \( t \). In the proportional catch population model \(^7\), the annual natural mortality rate of fish of age class \( i, i = 0, 1, \ldots \) is expressed as a proportion, \( M_i \), of the number of fish in this age class at the start of a year, so

\[
\begin{align*}
    n_{i+1, t+1} &= (1 - M_i)n_{it} - x_{it} \quad \forall \ i, t \quad (1a) \\
    n_{it} &= r_i \quad \forall \ t. \quad (1b)
\end{align*}
\]

Let \( f_i \) denote the proportional catch in year \( t \), namely the annual fishing mortality rate in year \( t \) expressed as a proportion of the total catchable stock at the start of year \( t \). Fishing activities tend to affect all age classes equally, apart from the impact of measures designed to protect young fish, for example minimum net mesh size, and so if \( Q_i \) is the catchable proportion of fish of age class \( i \)

\[
x_{it} = Q_i f_i n_{it} \quad \forall \ i, t. \quad (1c)
\]

In fish stocks models it is generally assumed that fish reach their maximum weight at age \( I \), with fish of age \( I \) and above having the same weight and mortality characteristics. Let \( n_{it} \) denote the number of fish of age \( I \) and above at the start of year \( t \), and let \( x_{it} \) denote the number of fish of this age class caught in year \( t \), and so

\[
n_{I+1, t+1} = (1 - M_{I-1})n_{I-1, t} - x_{I-1, t}. \quad (1d)
\]

If \( W_i \) is the mean weight at catch of fish of age class \( i \), the fishing yield, \( y_i \), in year \( t \) (by weight of catch) is

\[
y_i = \sum_i W_i x_{it}. \quad (2)
\]

Stock recruitment is widely considered to be related to the spawning stock biomass. If a proportion \( A_i \) of fish in age class \( i \) is mature, with mean spawning weight \( W_i \), the spawning stock biomass, \( s_i \), in year \( t \) is

\[
s_i = \sum_i A_i W_i x_{it}. \quad (3)
\]

For the Beverton–Holt recruitment function \(^11\), recruitment \( r_i \) in year \( t \) is given by

\[
r_i = D_1 s_i / (1 + s_i / D_2) \quad \text{for} \quad (4)
\]

where \( s_i \) is given by (3), and \( D_1 \) and \( D_2 \) are constants for a particular fish stock.

The multiperiod MIP model

Assuming fish price to be independent of catch, and annual fishing cost to be a linear function of catch, given by (2), Glen \(^3\) used the fish population model (1) to develop a multiperiod bioeconomic MIP model to determine the policy required to reach a specified target state from a given initial state. Since constraint (1c), representing the requirement that proportional catch \( f_i \) in year \( t \) must have the same value in each age class, is non-linear, this requirement is replaced by the approximation that \( f_i \) must lie in the same interval in each age class. By defining \( J, J > 0 \), equal width intervals for proportional catch \( f_i \) in year \( t \), with interval \( j, j = 1, 2, \ldots, J \), having lower bound \( B_{j-1} \), \( 0 < B_{j-1} < 1 \), and upper bound \( B_j \), \( 0 < B_j \leq 1 \), and defining a binary variable \( \delta_{jt} \) for this interval such that if \( \delta_{jt} = 1 \), then \( B_{j-1} \leq f_i \leq B_j \), this approximation can be represented by

\[
-x_{it} + B_{j-1} Q_i n_{it} + B_{j-1} Q_i N_j \delta_{jt} \leq B_{j-1} Q_i N_j \quad \forall \ i, t; \ j \geq 1 \quad (5a)
\]

\[
x_{it} - B_j Q_i n_{it} + B_j Q_i N_j \delta_{jt} \leq B_j Q_i N_j \quad \forall \ i, j, t \quad (5b)
\]

with

\[
\sum_{j=1}^J \delta_{jt} = 1 \quad (5c)
\]

where \( N_j \) is the maximum number of fish in age class \( i \).

The accuracy of the approximation given by (5) improves with decreasing interval width. To obtain a solution of the desired accuracy using a model based on a small number, \( J \), of proportional catch intervals, Glen \(^3\) proposed an iterative solution procedure in which the interval width is reduced at successive iterations, with the intervals for year \( t \) at iteration \( m, m \geq 1 \), having bounds \( B_m \), \( 0 \leq B_m \leq 1 \), \( j = 0, 1, 2, \ldots, J \), and forming a neighbourhood about the optimal interval at the previous iteration, namely iteration \( m - 1, m \geq 2 \).

Non-linear stock recruitment functions can be represented by piecewise linear approximations in this MIP model. For example, to represent the Beverton–Holt recruitment function (4) by a piecewise linear approximation with \( K - 1, K > 1 \), segments, assume that \( S_k \) is an upper bound on spawning stock biomass in any year, namely \( s_i \leq S_k, t = 1, 2, \ldots . \), and choose \( K \) biomass values \( S_k, k = 1, 2, \ldots, K \), with \( S_1 = 0 \) and \( S_1 < S_2 < \cdots < S_{K-1} < S_k \). Let \( R_k, t = 1, 2, \ldots, K \), be the annual number of recruits for spawning stock biomass \( S_k \) where, from (4)

\[
R_k = D_1 S_k / (1 + S_k / D_2).
\]

For the piecewise linear approximation, define variables \( u_{kl}, u_{kl} \geq 0, k = 1, 2, \ldots, K \), for year \( t \) such that

\[
\begin{align*}
    s_t &= \sum_{k=1}^K S_k u_{kt} \quad (6a) \\
    r_t &= \sum_{k=1}^K R_k u_{kt} \quad (6b)
\end{align*}
\]

where

\[
\sum_{k=1}^K u_{kt} = 1 \quad (6c)
\]