When to rush a ‘behind’ in Australian rules football: 
a dynamic programming approach

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In Australian rules football, points are scored when the ball passes over the goal line. Six points are awarded for a goal when the ball passes between the two centre posts, and one point for a ‘behind’, when the ball passes between a centre post and an adjacent outer post. After a behind, the defending team has a free kick from the goal line. It may be worthwhile, particularly in the closing stages of a game, for a defending team voluntarily to concede a behind, by themselves passing the ball between the two outer posts, either to avert the possibility of an imminent goal or to increase the probability of scoring a goal themselves. A dynamic programming model is used to analyse this situation.

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Introduction

When a game between two teams nears the end of the scheduled time, it may be advantageous for a team, particularly if it is losing, to change its strategy. A well known example is that of ‘pulling the goalie’ in ice hockey, treated by Washburn in a paper which demonstrates how a dynamic programming formulation can take account of all possible policies and not restrict consideration to a subset, as previous papers had done. End game tactics in other sports have been analysed, for example by Clarke and Norman on cricket and by Kohler on darts.

In this paper we consider the end game in Australian rules football, the major winter sport of the southern states of Australia, played between teams of 18 players on oval grounds (the same grounds used for cricket during the summer). A match is played in four quarters, each of 20 minutes plus about 10 minutes of extra time. Players can run with the rugby-shaped ball, but it is moved forward more quickly by kicking or punching it to a team-mate, and with no off-side rule, the game is fast. The scoring region consists of four upright posts. Kicking the ball between the two centre posts scores a goal worth 6 points, while the region between either centre post and the corresponding outside post scores a ‘behind’ worth one point. A ‘rushed’ behind is scored by a defender kicking, punching or carrying the ball over his own goal line, conceding a point in the hope of a possible territorial advantage. Draws are rare: a typical score might be anything between 50 and 150 points.

Description of model

We propose a simple model which is intended to capture the essential features of the game, particularly in regard to the decision whether or not to rush a behind. Such a model may be useful in generating and testing ideas before practical trials. To fix ideas, suppose we (our team) play up the ground, which we divide into seven parts, numbered as shown in Figure 1. We try to move the ball into areas 6 and 7 in which we can attempt to kick a goal. The probabilities of scoring 6 (for a goal) or 1 (for a ‘behind’) are the same as in Figure 1. We try to move the ball into areas 6 and 7 in which we can attempt to kick a goal. The probabilities of scoring 6 (for a goal) or 1 (for a ‘behind’) from area $i$ ($i = 6$ or 7), $(k_6^i + k_1^i = k_i)$. If a goal is scored (+6), the ball is bounced on the centre spot in area 4. If a ‘behind’ is scored (+1) or a kick at goal misses completely (no score) or the defending side rush a behind (−1) the defending side kicks from the goal area and has possession in area 3 with probability $\pi$.

When a team moves the ball, generally forward, it may maintain or gain ground (one area at a time) and keep or
lose possession with the following probabilities:

\[ p_1 \text{ keep ball, gain ground} \]
\[ p_2 \text{ keep ball, maintain ground} \]
\[ p_3 \text{ lose ball, maintain ground} \]
\[ p_4 \text{ lose ball, gain ground.} \]

\[ p_1 + p_2 + p_3 + p_4 = 1, \text{ and normally } p_1 < p_2 \text{ and } p_3 < p_4. \]

We take it that each of these transitions takes place between stages—decision epochs occurring at constant time intervals throughout the duration of the match.

To facilitate analysis, we suppose that a team will always move the ball or kick for goal when in area 7. In area 6 the team may either move the ball or kick for goal. In area 1 the team may either move the ball or rush a behind.

Let \( f_n(s, p, 1) \) be the probability of our winning the game, with \( n \) stages remaining, when our team leads by \( s \) points and we have the ball in area \( p \), under an optimal policy. ‘Optimal’ here means maximising the probability of winning the game.

Similarly, let \( f_n(s, p, 2) \) be the probability of our winning the game, with \( n \) stages remaining, when our team leads by \( s \) and the opposing team has the ball in area \( p \). Suppose also that the probabilities \( |p| \) \(|k|\) and \( \pi \) are the same for both teams.

Then

\[
f_n(s, p, 1) = \begin{cases} 
1 & \text{if } s > 0 \\
0 & \text{if } s \leq 0
\end{cases}
\]

\[
f_n(s, p, 1) = p_1 f_{n-1}(s, p + 1, 1) + p_2 f_{n-1}(s, p, 1) + p_3 f_{n-1}(s, p, 2) + p_4 f_{n-1}(s, p + 1, 2) \quad \text{for } p = 2, 3, 4, 5
\]

\[
f_n(s, 7, 1) = k^S_1[0.5f_{n-1}(s + 6, 4, 1) + 0.5f_{n-1}(s + 6, 4, 2)] + k^S_1[(1 - \pi)f_{n-1}(s + 1, 5, 1) + \pi f_{n-1}(s + 1, 5, 2)] + (1 - k^S_1)[(1 - \pi)f_{n-1}(s, 5, 1) + \pi f_{n-1}(s, 5, 2)]
\]

\[
f_n(s, 6, 1) = \max\{\text{move}: p_1 f_{n-1}(s, 7, 1) + \cdots + p_4 f_{n-1}(s, 7, 2) \}
\]

\[
k^S_1[0.5f_{n-1}(s + 6, 4, 1) + \pi f_{n-1}(s, 5, 2)]
\]

\[
f_n(s, 1, 1) = \max\{\text{move}: p_1 f_{n-1}(s, 2, 1) + \cdots + p_4 f_{n-1}(s, 2, 2) \}
\]

\[
\text{rush}: \pi f_{n-1}(s - 1, 3, 1) + (1 - \pi) f_{n-1}(s - 1, 3, 2)
\]

\[
f_n(s, 1, 2) = 1 - f_n(s + 1, 6 - p, 1) \text{ by symmetry, for the probability that we win when they have the ball in area } p \text{ and we lead by } s
\]

\[
= \text{the probability that they win when we have the ball in area } 6 - p \text{ and they lead by } s
\]

\[
= 1 - \text{probability that we tie or win when we have the ball in area } 6 - p \text{ and they lead by } s
\]

\[
= 1 - \text{probability that we win when we have the ball in area } 6 - p \text{ and they lead by } s - 1
\]

**Initial results**

A short computer program was written in BASIC to evaluate \( f_n(s, p, 1) \) and \( f_n(s, p, 2) \) for \(-25 \leq s \leq 25\) and \( p = 1–7\) for successive values of \( n \). The following parameter values were assumed:

\[
\begin{align*}
p_1 &= 0.3 & k^S_1 &= 0.2 & \pi &= 0.7 \\
p_2 &= 0.4 & k^S_2 &= 0.3 \\
p_3 &= 0.1 & k^S_3 &= 0.5 \\
p_4 &= 0.2 & k^S_4 &= 0.2
\end{align*}
\]

Although calculations were carried out for \( n = 1–50 \), the main purpose of the program run was to confirm that appropriate behaviour occurred at small values of \( n \) (near the end of the match). Table 1 shows when to rush a behind, indicated by a tick, depending on the score difference and the number of stages left. A tick indicates that in area 1, a defending team will increase its chances of winning by rushing a behind rather than moving the ball. For example, with ten stages left, a team which is two points ahead should rush a behind, a team which is one point ahead should not. Not surprisingly, when a team is up to 4 points behind, it can be worthwhile for it to rush a behind if there are enough stages left for it to have a chance of moving the ball the length of the ground and scoring a goal. At 5 points behind, this tactic is not worthwhile, as the point given up through the rushed behind makes it impossible to do better than tie.

When a team is between 2 and 6 points in front, it can be worthwhile for it to rush a behind because even if the opposing team gain the ball from the goal line kick off, they are further away from the goal line and less likely to score a goal.