Optimal production stopping and restarting times for an EOQ model with deteriorating items

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A perishable single item production-inventory system is studied in this paper. The objective is to describe a general model in which the production rate, the product demand rate, and the item deterioration rate are all considered as functions of time, and to discuss the optimal production stopping and restarting times which minimise the total relevant cost per unit time. In the general model, demand shortage is allowed, where some of the demand is lost and the rest is backlogged. Popular models, such as the pure inventory system and the zero shortage system, are shown to be special cases of our model. The conditions for a feasible stationary point to be optimal are given. The simplest cases with constant rates of production, demand and deterioration are discussed and shown as illustrative examples.

Keywords: EOQ; production; demand; deterioration; shortage

Introduction

EOQ inventory models have long been attracting considerable amount of research attention. For the last fifteen years, researchers in this area have extended investigation into various models with considerations of item shortage, item deterioration, demand patterns, item order cycles, as well as their combinations. For example, Goel\(^1\) studied the model with a variable deterioration rate, the rate is represented as a function of the inventory level and shortages are allowed with both partially lost and backlogged items. Sachan\(^2\) also discussed the cases of partially lost and backlogged items. Cheng\(^3\) extended the results of Smith\(^4\) to the case of a constant decaying rate for the deteriorating items. Murdeshwar\(^5\) extended the work of Deb and Chaudhuri,\(^6\) who studied the inventory replenishment policy for items having demand with linear trend and shortages. Goswami and Chaudhuri\(^7\) considered the model for deteriorating items with shortage and a linear trend in demand.

However, all this research is limited to the pure inventory replenishment situation. In other words, items are purchased or ordered in batches. Typical questions for such an inventory system are when and how much to order. The decisions on the optimal time and optimal quantity of ordering are based on the minimum total relevant cost per unit time.

On the other hand, research on production-inventory models has emphasised various production factors, such as the production size, the set up time, the production learning and forgetting effects, and the issues of machine breakdown, etc. Research findings in this area can be found in an abundance of publications. Cheng\(^8\) studied a model with demand-dependent unit production costs and solved the problem using geometric programming. Silver and Peterson\(^9\) studied the models where the setup and unit variable manufacturing costs are constant and independent of order quantities. Sule\(^10\) incorporated the effects of learning and forgetting in determining the economic production quantity in the model. Li and Cheng\(^11\) studied a production-inventory model with learning and forgetting effects on both production and setup. Haq et al.\(^12\) studied a production-inventory-distribution model for manufacturing, it was an integer programming based case study, where the optimal production and distribution quantities as well as the optimal inventory levels are discussed.

Product perishability is an important consideration for certain categories of inventory items. In the early 1980s, Nahmias\(^13\) and Rabbit\(^14\) reviewed the perishable inventory research findings up to the date. Most of these studies, including those in the last 15 years, have concentrated on pure inventory systems, involving different penalty cost structures\(^15\) or different demand patterns.\(^16,17\) Few publications have involved production. Srinivasan and Lee\(^18\) studied the perishability problem in production-inventory systems, where the facility is assumed to deteriorate during production. In our new model, the time dependent perishability, together with the time dependent production and demand rates, determines the inventory level, through the production-inventory stage, pure inventory stage and the shortage stage in each decision cycle. It is, therefore, a new product perishability study.

For generality, we assume that the production rate, demand rate and deterioration rate are all functions of time. The inventory level gradually builds up during the production period as the production rate is assumed to be
high enough to cover both item demand and deterioration. Due to the limitation of the inventory holding capacity and the requirement of regular machine maintenance, production needs to be stopped at a certain time while item demand and deterioration continue which are catered for by the accumulated inventory. We allow shortage to occur whereby part of the shortage is lost and the rest is backlogged for the next production cycle, that is, production is not necessarily restarted at the zero inventory level, or some predefined buffer inventory level. The objective is to determine the optimal production stopping and restarting times based on the minimum total relevant cost per unit time.

This is a single period decision making problem. In other words, the decision on the optimal production stopping and restarting times is made independently at the beginning of each cycle, based on the prevalent inventory level state which is the ending state of the previous cycle. Under such a structure, it is clear that the solution for a multi-period decision making is the same for the single period one.

As shown in Figure 1, a typical production cycle consists of four stages. In stage one, products are continuously produced and consumed until the inventory begins to build up; in stage two, the inventory level continuously increases until the production is stopped; in stage three, the inventory level decreases since production is stopped; and in stage four, shortage occurs and some demand is lost while the rest is backlogged. The production cycle begins at an initial inventory level which is the ending inventory level of the last cycle.

This paper presents the most general production-inventory model which considers the rates of production, demand and deterioration as time-dependent functions, and allow demand shortage to occur where a portion of the demand is lost while the rest is backlogged. Furthermore, other widely applied inventory systems, such as the pure inventory system without production and the production-inventory system where shortage is not allowed, are discussed as special cases of our model. Therefore, we can analyse the model with a view to identifying the properties for the optimal solutions in general.

The rest of the paper is organised as follows. The next section we give an illustrative example, which provides the original motivation of this study. Section 3 introduces the notations and definitions used throughout the paper. Section 4 presents the general model with a variable production rate, demand rate, deteriorating rate, and shortage, and gives the conditions under which the optimal production stopping and restarting times exist. Section 5 considers the special case with no item shortage. Section 6 shows that the pure inventory model is also a special case of our model with an infinite production rate. Finally, Section 7 summarises the major findings and presents some concluding remarks.

### An illustrative example

In the general production-inventory model discussed in this paper, the items are produced continuously during the production period, while demand and deterioration occur all the time. This model is initially conceived from an observation of a production line in a food factory, where a traditional Chinese light food is made on two major machines sequentially. On machine one, rice powder chips are made, and on machine two, the chips are pressed with sugar syrup to form a cake. Since machine two processes powder chips from machine one in batches, and it has a longer processing time than machine one, a work-in-process (WIP) inventory is built up for the continuous output of the production line. Product deterioration occurs as the fresh white chips begin to change colour in several minutes under normal temperature and need to be discarded if they are not treated with sugar syrup. It is then clear that the first machine needs to be stopped at some time to keep the WIP inventory in check. Thus the problem is to decide the optimal stopping and restarting times of machine one such that total production cost and WIP inventory cost is minimised.

In this case, since the machine is highly automatic, the production rate is a constant. The demanding rate, however, needs further elaboration. Under the observed manufacturing environment, the temperature on machine two is not well controlled. Therefore its production rate, which serves as the demanding rate of machine one, is observed as a random variable with a small variability. Ideally, if the production temperature can be precisely regulated, this demanding rate is expected to be a constant. Practically, with some minor adjustments to the production environment, we observe that as the temperature on machine two increases, it takes a shorter time to form and to press the cake, thus the demanding rate on the WIP inventory can be modeled as an exponential function of time $t$. Furthermore, the deteriorating rate of chips on the WIP inventory also depends on the production time. As the temperature increases following an increase in the inventory level, rice chips change colour faster. The deteriorating rate can again be

![Figure 1](image-url)