
Zhou and Lau\(^1\) claimed that the analysis of Jaber and Bonney\(^2\) contained mathematical errors which rendered the developed model invalid. This comment demonstrates that the production lot size model of Jaber and Bonney (JBM) produces satisfactory results. Mathematical models of the production lot size are reviewed and then three models: Zhou and Lau (ZLM),\(^1\) Jaber and Bonney (JBM),\(^2\) and Fisk and Ballou (FBM)\(^3\) are compared and their differences and similarities discussed.

**Background**

Learning suggests that the performance of a person or an organisation engaged in a repetitive task improves with time. This improvement is represented as a decrease in the cost of the product, but if the saving due to learning are significant, the effect on production time and hence inventory should also be significant. Factors contributing to this improved performance include more effective use of tools and machines, increased familiarity with operational tasks and the work environment, and enhanced management efficiency.

The use of the learning curve has been receiving increasing attention due to its application to areas other than in traditional learning. In general, learning is an important consideration whenever an operator begins production of a new product, changes to a new machine, or restarts production after some delay. Learning implies that the time (cost) needed to produce a product will be reduced as the individual (group) becomes more proficient with the new product or process.

Optimal lot size formulae are typically developed and used on the assumption that manufacturing unit costs are constant. With learning this approximation is true only when the production rate has stabilised.

Jaber and Bonney\(^4\) categorised publications on learning curve effects on the lot sizing problem into two groups. The first group includes those authors who studied the effects of learning on the economic manufacturing quantity (EMQ) with the assumption of instantaneous delivery.\(^5–9\) The second group includes those authors who studied the effects of learning on the EMQ with finite delivery rate.\(^2,3,7–19\)

Most researchers\(^5–20\) assumed that learning conforms to the power function formulation introduced by Wright.\(^21\) Fisk and Ballou\(^1\) studied the manufacturing lot-size problem under both the unbounded\(^21\) and bounded learning\(^22\) situations. Fisk and Ballou’s total holding cost expression was of a closed algebraic form. It was necessary, in their case, to resort to numerical techniques to find the production quantity, \(Q(t)\), for a given value of time, \(t\), using Newton–Raphson numerical search method. Then, for several determined values of \(Q(t)\) a numerical integration procedure was used, in particular Simpson’s rule.

Among the researchers who extended the work of Fisk and Ballou\(^1\) were Smunt and Morton\(^19\) who assumed that the holding cost is a function of the quantity produced, and Klasterin and Moinzadeh\(^18\) who allowed for continuous rather than discrete, lot sizes. Both researchers extended on the unbounded learning case.\(^21\) Jaber and Bonney\(^2\) are believed to be the first to extend on the bounded learning\(^22\) situation by presenting an efficient approximation for the total holding cost expression developed by Fisk and Ballou.\(^3\) This approximation simplified the solution of the lot-size problem with bounded learning by relying on one numerical search rather than two.

So far, the aforementioned researchers\(^2–20\) allowed for no shortages in developing their models. Jaber and Salameh\(^23\) extended the work of Salameh et al\(^18\) by allowing for shortages to be backordered. In a later paper, Jaber and Bonney\(^24\) assumed a case where it is sometimes possible that the improvement in the production rate is slow due to the complexity of the task performed, and it is possible for the demand rate to exceed the production rate for a portion of the production cycle. This case was referred to as intracycle backorders. In a recent paper, Zhou and Lau\(^1\) extended the work of Jaber and Bonney\(^2\) by allowing shortages to be backordered. This was done by incorporating the work of Jaber and Salameh\(^23\) into that of Jaber and Bonney.\(^2\) In addition, Zhou and Lau\(^1\) claimed that the analysis of Jaber and Bonney\(^2\) contained mathematical errors which rendered the developed model invalid, where both models are approximate representations of that of Fisk and Ballou.\(^3\)

This paper demonstrates that the production lot-size model of Jaber and Bonney (JBM) produce satisfactory results. Mathematical models of the production lot size are reviewed and then three models: Zhou and Lau\(^1\) (ZLM),
Jaber and Bonney\(^2\) (JBM), and Fisk and Ballou\(^3\) (FBM) are compared and their differences and similarities discussed.

The rest of the paper is organised as follows. The next section is the introduction. The following section describes the mathematical modelling of ZLM and JBM and after that we provide illustrative numerical examples and sensitivity analysis for ZLM and JBM. The penultimate section compares ZLM and JBM with FBM and the last section presents a summary and conclusions.

**Introduction**

The bounded learning curve function was proposed by de Jong.\(^2\) This function includes both a fixed and a variable component. The fixed component represents the minimum task time per unit produced, while the variable time is subject to learning. The bounded power function formulation represented by de Jong\(^2\) as

\[
T_j = T_{1i}M + (1 - M)T_{1i}^{-b}
\]

where \(T_j\) is the time to produce the \(j\)th unit in cycle \(i\), \(T_{1i}\) is the variable component of the time to produce the first unit in the \(i\)th cycle; where \(T_{1i} = T_{1i}(1 + \sum_{n=1}^{j-1} q_n)^{-b}\) and \(q_n\) is the quantity produced in production cycle \(n\), \(M\) is the incompressibility factor \((0 \leq M \leq 1)\), and \(b\) is the learning slope, computed as \(b = -\log \phi / \log 2\) with \(\phi\) as the learning rate. Figure 1 illustrates the behaviour of de Jong’s learning curve for different values of incompressibility factor \(M\). For a value of \(M\) equals to zero, (1) conforms to that of Cherrington et al.\(^2\) ie \(T_j = T_{11}^{-b}\), which is a modified version of that of Wright.\(^2\)

In any production cycle \(i\), define \(T_{1i}\) as the production time to produce \(q_i\) units and accumulate a maximum inventory of \(Z_i\), \(t_{2i}\) as the time to deplete \(Z_i\) units at a constant demand rate of \(r\) units per unit of time, and \(t_{ci}\) as the cycle time, ie \(t_{ci} = q_i/r = t_{1i} + t_{2i}\). Figure 2 illustrates the effect of learning on build up inventory.

**Mathematical modelling of ZLM and JBM**

Zhou and Lau\(^1\) production lot-sizing model (ZLM) presented a total cost per unit time function. \(TCU(q_i, q_{1i})\), of the form

\[
TCU(q_i, q_{1i}) = \frac{K}{q_i} + d_m r + L \left( T_{1i}Mr + \frac{1 - M}{1 - b}rT_{1i}q_i^{-b} \right)
\]

\[
+ \frac{h r}{q_i} \left( \frac{1}{2r} (q_i - q_{1i})^2 (1 - rMT_{1i}) \right)
\]

\[
+ \frac{1 - M}{1 - b}q_i \left[ (q_i - q_{1i})q_{1i}^{-b} - \frac{q_i^{2-b} - q_{1i}^{2-b}}{2 - b} \right]
\]

\[
+ \frac{s r}{q_i} \left( \frac{1}{2r} q_i^2 (1 - rMT_{1i}) - \frac{1 - M}{2 - b} T_{1i}q_{1i}^{-2b} \right)
\]

where \(K\) is the set cost, \(d_m\) is the material cost per unit item, \(L\) is the labour cost per unit of time, \(h\) is the inventory holding cost per unit per unit of time, \(s\) is the shortage cost per unit per unit of time, \(q_i\) is the manufactured quantity in cycle \(i\), and \(q_{1i}\) is the portion of \(q_i\) that covers for the backordered items. All other parameters in (2) have been defined at an earlier stage in this paper.

![Figure 1](image1.png)

**Figure 1** The behaviour of de Jong’s\(^2\) learning curve for different values of incompressibility factor \(M(T_{11} = 10, 1 \leq j \leq 50, b = 0.1)\).

![Figure 2](image2.png)

**Figure 2** Variation in inventory level with learning considerations.