Evaluating public school district performance via DEA gain functions

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Existing measures of input allocative efficiency may be biased when estimated via data envelopment analysis (DEA) because of the possibility of slack in the constraints defining the reference technology. In this paper we derive a new measure of input allocative efficiency and compare it to existing measures. We measure efficiency by comparing the actual outputs of a decision-making unit relative to Koopmans’ efficient subset of the direct and indirect output possibility sets. We estimate the existing measures and our new measure of input allocative efficiency for a sample of public school districts operating in Texas.

Keywords: data envelopment analysis (DEA); Farrell and Zieschang gain functions; production theory; school efficiency

Introduction

In the three decades following publication of the ‘Coleman Report’ various states and the federal government implemented reforms to try to improve student achievement. Although these modern day efforts stem from citizens’ and policy-makers’ desires to improve school accountability, Rapple2 finds that government concern over school efficiency dates from at least 1862–1897, when governmental grants to English and Welsh elementary schools depended on how well students answered questions administered by Her Majesty’s Inspectors. Since many studies (Hanushek3 provides a review) find little significant correlation between school resources and school outcomes, researchers began to examine the efficiency with which schools use resources. Bessent et al4 provide an early study of school efficiency using data envelopment analysis (DEA).

The purpose of this paper is two-fold: first we review and critique the educational efficiency literature as it has progressed over the last two decades; second we propose a new measure of input allocative efficiency and estimate and compare it to existing measures for a sample of public school districts operating in Texas in the United States. A decision-making unit is said to allocate inputs efficiently when the ratio of marginal products (the marginal rate of substitution between inputs) equals the ratio of input prices. When inputs are allocated inefficiently it is possible to either reduce costs while holding output constant, or expand outputs while holding costs constant, by appropriately reallocating inputs. Given the public’s concern about the lack of correlation between public school outcomes and public school spending, the identification of input allocative efficiency is an important issue for policy makers.

Early attempts at estimating the allocative efficiency of schools involved specification of a single output–multiple input production function and estimation of the marginal effects of school inputs via regression techniques. Given input prices and estimated marginal products, one can then make inferences on how resources should be reallocated. Hanushek3 provides an excellent overview of the issues involved in estimating educational production functions. Because schools typically produce multiple outputs, researchers subsequently turned to other techniques, such as the estimation of a cost function for educational outcomes or DEA to evaluate the technical efficiency of schools (Callan and Santerre5 and Färe et al6). Instead of using a production function to obtain the marginal rate of substitution between inputs, Grosskopf et al7 use stochastic techniques to estimate the marginal rate of substitution between inputs as the ratio of the derivatives of the input distance function with respect to inputs. Comparing the marginal rate of substitution between administrators and teachers relative to the ratio of administrator salaries to teacher salaries, Grosskopf et al7 find that allocative inefficiency could be reduced by increased competition between school districts.

Recently, Grosskopf et al8 introduced a gain function for public school districts based on Farrell9 efficiency measures and Shephard10 multi-output production functions. Their gain function measures the output expansion that could occur if school districts were able to optimally reallocate inputs while maintaining a given budget. They attributed the loss of educational output to input regulatory restrictions which constrain a school district’s choice of inputs. Examples of such restrictions include minimum ratios of teacher to students and minimum ratios of teacher aides to teachers.
Calling the gain function the input allocative efficiency measure, Fukuyama et al.\textsuperscript{11} extend Grosskopf et al.\textsuperscript{8} static model into a dynamic one and then study productivity. As is well known, Farrell efficiency measures compare a decision-making unit’s output-input combination to the input isoquant when measuring input efficiency or to the production possibility frontier when measuring output efficiency. When estimated via DEA, the Farrell measures of efficiency may allow slack in the constraints defining the technology so that it is possible to expand at least one output at no cost.

In this paper we extend the Grosskopf et al.\textsuperscript{8} gain function to a class of DEA efficiency measures that do not allow slack in the output constraints defining the technology. The input allocative efficiency measure we propose compares the efficiency of decision-making units (school districts) relative to Koopmans’s\textsuperscript{12} efficient subset of the direct and indirect output possibility sets and is based on Zieschang’s\textsuperscript{13} extended Farrell efficiency measure. Similar to Grosskopf et al., our gain function (which we call the Zieschang gain function) measures the expansion in output that would be feasible if the school district optimally reallocated inputs, holding the available input budget constant.

Related to our gain function, a referee called our attention to contributions by Cooper et al.\textsuperscript{14} and Seiford and Zhu\textsuperscript{15} who consider slack within a weighted additive model framework (see also Charnes et al.\textsuperscript{16} for the additive model). Cooper et al.\textsuperscript{14} construct slack-based indexes, while Seiford and Zhu\textsuperscript{15} obtain weights by the Delphi and AHP techniques. In contrast, our gain function is constructed using duality concepts, which allows economic interpretations.

Although our focus is on the correct way to measure the efficiency of schools, it is important to recognize that efficiency measures have meaning only to the extent that school outputs and inputs are measured correctly, an issue for which there is much disagreement. Bradley et al.\textsuperscript{17} provide a review of fourteen studies of school efficiency and alternative ways of defining inputs and outputs. We follow the work of Hanushek and Taylor\textsuperscript{18} and Grosskopf et al.\textsuperscript{7,8,19} in defining school outputs and inputs, a discussion we take up again in the empirical section.

In the next section we define the production technology for school districts, trace the development of school efficiency studies and introduce our new measure of input allocative efficiency. In the empirical section of the paper we estimate each of the technical and allocative efficiency measures, including our new measure of input allocative efficiency for a sample of Texas school districts. The final section summarizes our findings.

**Background and theoretical framework**

In this section we provide a theoretical framework that allows us to trace the history of school efficiency studies. We show how each of the functional representations of the school production technology can be used to estimate the efficiency with which schools allocate resources and we propose a new measure of input allocative efficiency. Table 1 provides a summary of each of the measures of efficiency that are developed in this section, a sampling of researchers who have used the various approaches to estimate school efficiency, their estimation method, and some of their findings.

We begin our discussion by defining the production technology that is the foundation for each of the efficiency measures that we summarize and propose. Let \( x \in \mathbb{R}^n_y \) denote a vector of inputs and let \( y \in \mathbb{R}^m_x \) denote a vector of outputs. Input prices are represented by \( w \in \mathbb{R}^n_x \) so that actual costs are \( \bar{w}x = c \). In the empirical section we partition \( x \) into variable inputs (\( x_v \)) that school administrators can exercise discretion over and fixed inputs (\( x_f \)) that school administrators must take as given, including students’ past test performance. For ease of exposition, we defer this partition until the empirical section.

The production technology is the set of all feasible inputs \( (x) \) that can produce outputs \( (y) \) and is defined as

\[
T = \{(x, y) : \text{factors of production } x \text{ can produce outputs } y\}
\]

(1)

We assume that the technology satisfies the property of no free lunch, is closed and convex, and exhibits strong input and output disposability. Strong input and output disposability implies that if \( (x^1, y^1) \in T \) and \( (x^1, -y^1) \leq (x^2, -y^2) \) then \( (x^2, y^2) \in T \). Relative to \( T \) and assuming one output, the production function is

\[
f(x) = \max\{y : (x, y) \in T\}
\]

(2)

Eberts and Stone\textsuperscript{20} estimate a school production function for unionized and non-unionized schools and find that unionized schools outperform non-unionized schools for the majority of students who are in the middle of the achievement distribution, but that non-unionized schools outperform unionized schools for students who are in the tails of the achievement distribution.

Assuming that \( f(x) \) is differentiable and defining the marginal products of inputs \( i \) and \( j \) as \( f'_i(x) \) and \( f'_j(x) \), inputs are allocated efficiently if \( [f'_i(x)/f'_j(x)] = w_i/w_j \). In reviewing the literature on educational production functions for developing countries, World Bank researchers Pritchett and Filmer\textsuperscript{21} point out that much of the empirical work finds that the marginal product per dollar for non-teaching inputs, like books and desks, are 10–100 times the marginal product per dollar for teachers. Since this evidence suggests a large misallocation of school resources, they argue that policies that shift school input choice toward parents and away from teachers ‘can sometimes lead to enormous gains in the cost effectiveness of schools’ (Pritchett and Filmer\textsuperscript{21}, p. 223).

Since publication of the study *Equality of Educational Opportunity* by James S. Coleman et al.\textsuperscript{1} numerous researchers have estimated educational production functions. Hanushek\textsuperscript{5} summarizes and synthesizes 147 studies of...