Transverse stability of an impact-ionization front in a Si $p^+ - n - n^+$ structure

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The transverse stability of an impact-ionization front in a large-area silicon $p^+ - n - n^+$ structure is studied. An analytical model allowing for simultaneous motion of the ionization front and displacement of the majority carriers from the nondepleted part of the $n$ base is proposed. The stability of a planar front is investigated, the growth increments are calculated, and the physical mechanisms of instability are indicated. A criterion is formulated for quasistable propagation of a wave. © 1997 American Institute of Physics. [S1063-7826(97)01104-6]

1. INTRODUCTION

The excitation of an impact-ionization wave is a unique, with respect to its rate, and at the same time very universal “nonoptical” mechanism for modulating the conductivity of a semiconductor structure — diode, transistor, or thyristor. In a recent report we showed that the propagation of a wave in a large-area $p^+ - n - n^+$ structure is accompanied by an increase in the transverse perturbations of the position of the front. Long-wavelength modes, whose buildup time is close to the time over which the ionization front (IF) traverses its own length, play the dominant role. The development of an instability can result in the formation of local switching channels with characteristic transverse size $W$. The nonuniform dynamics of the impact-ionization front, including a stage of formation of local switching channels, was described in Ref. 6. The transverse instability of the IF is a fundamental factor limiting the possibility of diode structures which are switched with the aid of an impact-ionization wave.

This paper continues the theoretical investigations of the stability of impact-ionization fronts. It considers the problem of the stability of the front in the most important case in which an undepleted quasilinear region is present in the $n$ base. The propagation of the IF in such a structure is accompanied by the displacement of carriers from the quasineutral region. As a result, the dynamics and stability of the IF should be studied together with the motion of the boundary between the space-charge region and the quasineutral region. An analytical model of the propagation of an impact-ionization wave in a Si $p^+ - n - n^+$ structure is proposed and its transverse stability is analyzed.

2. DESCRIPTION OF THE MODEL

We assume that initially the impact-ionization wave propagates uniformly over the entire area of the device. We studied a structure whose base contains, at least initially, an undepleted region, i.e. the condition $W > [2 \varepsilon \varepsilon_0 u q N_d]^{1/2}$ is satisfied (here $u$ is the voltage applied to the structure, $N_d$ is the density of donors in the $n$ base, $q$ is the electron charge, $\varepsilon$ is the permittivity of the semiconductor, and $\varepsilon$ is the permittivity of free space). Below we call such a structure a long-base structure, in contrast to the similar TRAPATT diode structures, in which the depleted region occupies the entire $n$ base.

Three regions can be distinguished in the $n$ base (Fig. 1): a neutral plasma region (NR); a space-charge region (SCR), including a completely depleted region, the IF itself, and the space-charge layer behind the front; and, a region of dense plasma behind the front. The characteristic carrier densities for the case of a Si diode are presented in the caption in Fig. 1. We assume below that the ionization coefficients are the same for electrons and holes and we use a step approximation for the impact-ionization coefficient as a function of the electric-field intensity $E$: $\alpha(E) = \alpha_0 H(E - E_a)$, where $H$ is the unit step (Heaviside) function, $E_a$ is the impact-ionization threshold, and $\alpha_0$ is a model parameter, which represents the saturated value of the impact-ionization coefficient. We disregard the impact ionization in the NR. The field in the region of the dense plasma behind the front is assumed to be weak and the distribution of the field in the SCR is assumed to be trapezoidal.

The position of the IF $x_f(y, z)$, together with the position $d(y, x)$ of the boundary between the SCR and the NR, completely determine the distribution of the field $E(x, t)$ in a given section of the structure $(y = \text{const}, z = \text{const})$. The equations of motion for the variables $x_f(y, z)$ and $d(y, z)$ have the form

$$\frac{\partial x_f}{\partial t} = \frac{1}{\tau_f} l_f(u, x, d),$$

$$\frac{\partial d}{\partial t} = -\nu_n(E_0(u, x, d)),$$

where

$$\tau_f = \frac{1}{\alpha \nu_s} \ln \frac{N}{n_0}, \quad N = \frac{\alpha_0 \varepsilon_0 E_a}{q},$$

$$\nu_n(E) = \frac{\mu_0 E}{1 + E/E_s}.$$
Here \( v_i \) is the unsaturated carrier velocity, \( N \) is the density of the nonequilibrium electron-hole plasma behind the IF, \( n_0 \) is the carrier density in the SCR in front of the IF, \( E_s = v_i / \mu_n \approx 3 \times 10^6 \) V/cm is the characteristic field in which the drift velocity saturates, and \( \mu_n = 1500 \) V · cm²/s is the weak-field electron mobility. A direct calculation shows that the size \( l_f \) of the ionization region and the field \( E_0 \) in the quasineutral region are given by the expressions

\[
\begin{align*}
l_f &= \frac{e e_0}{q N_d} \left( \frac{u}{W - x_f} - E_a \right) + \frac{1}{2} \left( W - x_f - \frac{d^2}{W - x_f} \right). \\
E_0 &= \frac{u}{W - x_f} - \frac{q N_d (W - x_f - d)^2}{2 e e_0} \left( \frac{W}{W - x_f} \right).
\end{align*}
\]

Equation (1) describes the propagation of the IF in a model proposed in Refs. 6 and 8. The derivation of this equation depends on two universal laws of propagation of impact-ionization waves and streamers. The front velocity \( v_f \) is proportional to the size \( l_f \) of the impact-ionization region; the displacement time \( \tau_f \) of the electric field out of the ionization region is determined by the transport time of the space charge associated with the front through this region.

The velocity of the boundary of the NR equals the velocity of the electrons displaced to the \( n^+ \) emitter. The minority carriers (holes) are then extracted into the SCR. The density \( p \) of the minority carriers is low compared with \( N_d \). As a result, their contribution to the space charge in the SCR is very small. In Eq. (2) an approximation of the velocity \( v_n(E) \) is used — the carrier velocity as a function of the electric field strength in Si proposed in Ref. 10.

Equations (1)–(4) must be supplemented by the Kirchhoff equation

\[
u = V(t) - q N_d R S \left( \frac{d x_f}{dt} \right),
\]

which relates the average (over the area) velocity of the front with the voltage on the structure \( V(t) \) is the source voltage, \( R \) is the load resistance, and \( S \) is the area of the structure. In Eq. (5) the relation of the current density \( j \) in a given section of the structure with the velocity of the front is taken into account. This closed system of equations completely describes the dynamics of a planar IF and an IF that is perturbed by long-wavelength \( (\lambda > W) \) fluctuations. The condition that the total current is continuous along the \( x \) direction holds automatically for Eqs. (1)–(4).

3. DYNAMICS OF A PLANAR IONIZATION FRONT

In the present case of a long-base structure, the one-dimensional dynamics of the IF exhibits several characteristic features which have no analogs for structures such as TRAPATT diodes. Two new phenomena can be distinguished: retardation of the IF as a result of a redistribution of voltage between the SCR and the NR, whose most extreme manifestation is stopping of the wave, and avalanche breakdown of the NR. We shall give only a brief description of these effects, which is necessary for further understanding of the results of the stability analysis.

1. Retardation of the IF. Differentiating Eqs. (1) and (2) with respect to time shows that the sign of the acceleration of the IF is determined by the sign of the right side of the expression

\[
\left( \tau_f + q N_d R S \frac{\partial l_f}{\partial u} \right) \frac{d v_f}{dt} = \frac{\partial l_f}{\partial u} \frac{d V(t)}{dt} + \frac{d v_n - \bar{d} v_f}{W - x_f},
\]

where

\[
\bar{d} = -(W - x_f) \frac{\partial l_f}{\partial x_f} = (W - x_f) - \frac{e e_0 E_{\max}}{q N_d}, \quad \frac{\partial l_f}{\partial u} > 0.
\]

The meaning of the quantities \( \bar{d} \) and \( E_{\max} \) is clear from Fig. 1. For sufficiently large \( W \), the quantities \( d \) and \( \bar{d} \) are close to one another, while \( v_f \) can be several times greater than \( v_n \). As a result, for \( V(t) = \text{const} \) the velocity of the IF decreases as the front propagates, and it is possible for the wave to stop. The physical mechanism of the stopping of the wave is connected with the fact that the growth of the field occurring in the NR during the motion of the IF results in a voltage drop across the SCR.

2. Breakdown of the quasineutral region. Since \( v_{n_f} > v_i \) and \( v_n < v_i \), the IF always overtakes the receding boundary of the quasineutral region. The SCR decreases in size and the field in the NR increases. The impact-ionization threshold can be reached before the majority carriers are completely removed from the \( n \) base, as a result of which avalanche breakdown of the NR ceases. In the simplest case when the current in the external circuit is fixed, \( v_f = \text{const} \), and the carrier velocity in the NR equals the saturated value \( v_i \), the condition that the IF traverses the entire \( n \) base before \( E_0 \) reaches \( E_a \) can be represented in the form