Modeling the wave parameters of a narrow slotted transmission line based on a superconducting film

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An analytical expression for calculating the wave parameters of a narrow-slot transmission line is found in the quasistatic approximation. A two-fluid model is used for analyzing processes in a superconducting film of thickness comparable to the London penetration depth. The wave parameters of the slotted line are calculated under assumptions about the current distribution near the edges of the slot which are analogous to those used previously for analyzing microstrip and coplanar lines. © 1997 American Institute of Physics. [S1063-7842(97)02009-6]

INTRODUCTION

High-temperature superconducting (HTSC) slotted transmission lines have recently attracted considerable attention. This has happened, in particular, because of the combined use of HTSC slotted line with a Josephson junction in microwave squids. In order to impedance match this type of system, it is necessary to have a minimum wave impedance in the slotted line. This is achieved by reducing the slot width to micron sizes. The existing analytical descriptions of the parameters of slotted lines are restricted to slot widths exceeding 2% of the substrate thickness. For slots that are narrow enough, it is possible to use a quasistatic approximation for calculating their linear parameters and to obtain simple analytical expressions. Because of the small, but finite resistance of the conductor, HTSC transmission lines have Ohmic losses, and these can be significant if the slotted line is sufficiently narrow. The method employed in this article has already been used to calculate the wave parameters in HTSC microstrip and coplanar transmission lines. The computational results are in good agreement with experimental data, which indicates that this approach can be applied to other types of flat transmission lines. In this paper we present analytical expressions which make it possible to calculate the wave parameters of HTSC slotted lines with given parameters.

QUASISTATIC CALCULATION OF THE LINEAR PARAMETERS OF A NARROW SLOTTED LINE

The transverse cross section of the line is shown in Fig. 1a. The substrate has a thickness h and width D. A superconducting film of thickness d is deposited on the substrate and the slot width is w. The effective permittivity of the line, which determines the phase velocity of a wave in a line with an ideally conducting coating, is taken to be

$$\varepsilon_{\text{eff}} = \frac{\varepsilon_r + 1}{2},$$

where $\varepsilon_r$ is the relative permittivity (dielectric constant) of the substrate material.

A comparison with electrodynamic calculations of $\varepsilon_{\text{eff}}$ for slotted lines shows that as $w/h$ approaches zero, the actual value of $\varepsilon_r,\text{eff}$ approaches that given by Eq. (1) (Fig. 2a).

Let $L_1$ and $C_1$ denote the linear and capacitance per unit length of the line (in the equivalent circuit, the inductances are in series, while the capacitances are in parallel). We use the fact that

$$L_1 = \left( Z_0 \sqrt{\varepsilon_{\text{eff}}}/c \right), \quad C_1 = \sqrt{\varepsilon_{\text{eff}}} \left( Z_0 c \right)^{-1},$$

where $Z_0$ is the wave impedance of the line and $c$ is the speed of light in vacuum.

In order to calculate the capacitance per unit length of a slot in a shield of width D, we have used the approximation of partial capacitances,

$$C_1 = \varepsilon_0 \left[ (\varepsilon_r - 1) - F(k_1) + F(k_2) \right],$$

where $\varepsilon_0$ is the vacuum permittivity. $F(k_1)$ gives the contribution of the substrate to the slot capacitance, and $F(k_2)$ gives the contribution of the surrounding space.
To find \( F(k_1) \) and \( F(k_2) \) we used a conformal mapping\(^7,8\) which gives the following expressions for the \( F(k_i) \):

\[
F(k_i) = \frac{K(k_i^2)}{K(k_i)},
\]

where \( K(k) \) is the complete elliptic integral of the first kind, and \( k_i^2 = 1 - k_i^2 \).

For \( w \ll h \ll D \) we have \( k_1 = \pi w/h \) and \( k_2 = w/D \). For the subsequent transformation, we used the approximation\(^3\)

\[
F(k_i) = \pi^{-1} \ln(2 \cdot (1 + \sqrt{k_i^2})/(1 - \sqrt{k_i^2}))
\]

for \( 0 \ll k \ll 0.707 \).

Expanding the argument of the logarithm in Eq. (5) in a series in the small parameter \( k_i \), we obtain a simple analytical expression for the capacitance per unit length of the slotted line and then, using Eqs. (1) and (2), an expression for \( Z_0 \):

\[
Z_0 \sqrt{\mu_0/\varepsilon_0((\varepsilon_r + 1)/2)^{0.5}} \pi((\varepsilon_r - 1)
\times \ln(16h/(\pi w)) + 2 \ln(4D/w))^{-1}.
\]

It is important to note that this approximation is valid for \( D < \lambda_0 \), where \( \lambda_0 \) is the free-space wavelength. For \( D \gg \lambda_0 \), it is necessary to substitute \( D^* = \lambda_0/2 \) in Eq. (6). The wave impedances \( Z_0 \) calculated using Eq. (6) are in good agreement with the data of Ref. 4 for \( w/h > 0.02 \) (Fig. 2b).

### THE CONTRIBUTION OF THE SUPERCONDUCTING LINE

When deriving the true values of the linear parameters for an HTSC slotted line, it is necessary to add the kinetic inductance \( L_1^{(k)} \) per unit length to \( L_1 \) and to account for the resistance \( R_1 \) owing to the real component of the complex conductivity of the superconducting film.

The sequence for finding the losses in the transmission line is as follows: initially the finite conductivity of the film is neglected and the boundary value problem is solved, in this case, in a quasistatic approximation. Then it is assumed that including \( L_1^{(k)} \) and \( R_1 \) does not change the distribution of the fields or the magnitude of the currents. Using the current distribution obtained under the assumption that the film does not have finite conductivity, we then find values of \( L_1^{(k)} \) and \( R_1 \) which are, therefore, determined by the surface current distribution and by the surface impedance of the superconducting film.

The complex conductivity of the superconductor can be written in the approximation of a two-fluid model as follows:

\[
\sigma = \sigma_1 - j\sigma_2.
\]

The active part \( (\sigma_1) \) originates in the normal electronic conductivity and is the cause of the losses in the line. The reactive part \( (j\sigma_2) \) originates in the inertia of the dissipationless motion of the superconducting carriers. This part of the conductivity is related to the London penetration depth \( \lambda_L \) by

\[
\sigma = (\omega \mu_0 \lambda_L^2)^{-1},
\]

where \( \omega \) is the angular frequency and \( \mu_0 \) is the vacuum magnetic permeability.

An expression for \( j(x) \) was found from a quasistatic analysis of current flow in the transmission line with the use of conformal mapping.\(^5,6\) In order to avoid a divergence in the integral of the square of the surface current density near the edges of the film, the transformation described in Ref. 5 was carried out. The resulting distribution has the form

\[
j_j(x) = \begin{cases} 
\frac{4}{\pi} \frac{1}{\sqrt{w\lambda_\perp}}, & w/2 \leq |x| < w/2 + \lambda_\perp, \\
\frac{1}{\pi} \frac{1}{\sqrt{2x/w}} & |x| \geq w/2 + \lambda_\perp,
\end{cases}
\]