Self-consistent development and fractal structure of leader discharges along a water surface

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Ideas regarding the development of single- and multichannel leader discharges over water surfaces are generalized on the basis of experimental data. The Ohmic conductivity of water is found to be manifested in the self consistency of their dynamics and fractal structure. The fractal dimensionality of a single-channel leader discharge is found to be $0.96 \pm 0.05$ and that of a multichannel discharge to be $1.85 \pm 0.05$. Mechanisms are proposed for the branching of leader discharge channels and for the development of bifurcations from branches and channels.

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Single- and multichannel leader discharges over water surfaces similar to Lichtenberg figures have been studied by the author. The Ohmic conductivity of water was found to cause a nonlinearity in the $R(t)C$-discharge circuit and a one-to-one correspondence between the channel lengths and the current passing through them, and that, as a whole, the discharge develops in a self-consistent manner. Given the multiplicity of elements in the structure of discharges of this type, the self consistency of their development must also have an effect on their structure. This paper is devoted to a study of the structure of leader discharges over water surfaces and its relationship to the dynamics of their development based on the earlier experimental work.

EXPERIMENTAL CONDITIONS

The experimental setups of Refs. 1 and 2 are sketched in Fig. 1a and 1b. They consisted of a cell with water (1), a storage capacitor $C = 0.1 \mu F$ (2), an anode in the form of a wire 0.0075 cm in diameter or a tip with that radius (3), and a cathode (4). A detailed description of the setup and diagnostic methods is given in the earlier papers. Both experiments used the same range of initial voltages on the capacitor ($U_0 = 3–6$ kV), conductivity of the water ($\approx 1 \times 10^{-4} \Omega^{-1} \text{ cm}^{-1}$), length of the air gap between the anode and water surface (0.1–0.3 cm, depending on $U_0$), and depth of immersion of the cathode below the water surface ($\approx 0.3$ cm). Only the types of cathode were different. In Ref. 1 a one dimensional cathode consisting of a wire 8 cm long with a diameter of 0.16 cm was used (Fig. 1a) and in Ref. 2, a flat, circular cathode with a diameter of 7 cm (Fig. 1b). This resulted in a substantially different distribution of the electric field at the water’s surface. In the first case a single-channel discharge develops and in the second, a multichannel discharge (Fig. 2). In both cases, however, the character of the $R(t)C$-circuits and their nonlinearity were determined by the resistance of the layer of water above the cathode and their electrical characteristics varied with time in a qualitatively similar fashion.

After a voltage $U_0$ was applied to the anode, a pulsed corona developed in the air at its tip. As the corona lengthened and reached the water’s surface, the current in the circuit rose and the discharge channel contracted and grew toward the surface of the water. The potential jump at the water’s surface at the time the current channel reached the water initiated the formation of several radial leader channels in the plasma spot on the water. As they developed, the area of the spot increased, as well as that of the layer of water above the cathode through which the current flowed. This reduced the resistance of the layer and caused the current to rise further. As the current rose, and, therefore, the power dissipated in the circuit increased, the resistance of the discharge channel in the air and the resistance of the leader channels fell; this reduced the voltage drop across them and sustained the potentials at their tips, enabling them to move onward. This, in turn, ensured a further increase in the current. Naturally, in the case of the one-dimensional cathode this occurred only for the channel that developed above it, except during the beginning of the discharge, when the conditions for all the channels were similar. This sort of one-to-one correspondence between the channel lengths $l$ and the current $i$ in them, which can be regarded as a positive feedback, was maintained until the potentials of the tip and the water below it were no longer equal owing to the rise in the voltage drop across the leader channel. At that time $t_M$, the leader stopped and its length and the current through it reached their peak values ($l_M$ and $i_M$). From this time on, the

FIG. 1. A sketch of the experimental apparatus: (a) single-channel and (b) multichannel leader discharges.
The final structure of a multichannel discharge consists of 3–6 initial channels, their branches, and their subbranches and those of the channels.\(^2\) In accordance with an analysis of the dynamics of the channel development in Ref. 2, in this case two channels \(I\) and \(2\) (sectors \(\varphi_1\) and \(\varphi_2\) of Fig. 2\(b\) and 2\(c\)) appear synchronously and then channel 3 (sector \(\varphi_3\)), completing the formation of a stable initial electrostatic structure. Here the first two initial channels developed radially in one half plane and the third channel, in the other half plane. This is related to the axial symmetry of the anode field at the water’s surface and its dominant role in the initial development of the channels. The first bifurcations of the channels are, respectively, \(\varphi_1 \approx \varphi_2 \approx 80\,\text{deg}\) and \(\varphi_3 = 140\,\text{deg}\). However, the angles of the later bifurcations are close to 40–50 deg for all the channels, and the final structure of the branches of the first two initial channels occupies a sector \(\varphi_1 + \varphi_2 > 180\,\text{deg}\). This all indicates that, with distance from the anode, the local field in front of the tips of the channels affects the discharge structure; this field is random and causes the differences in the channel development. Therefore, the discharge structure is formed by the motion of the initial channels and their branching as they move under the influence of the central and local electric fields.

Channel branching is fundamentally intrinsic to extended discharges in air, but its mechanism has apparently not been specially studied. Bazelyan and Razhanski\(^{11}\) have considered the branching of a streamer as the development, with time, of a second streamer on its side when the radial field strength exceeds 150 kV/cm and the effective potential, 10 kV. In our case, these magnitudes are not so high and photographs show that the branches of the final bifurcations have roughly the same lengths, which suggests that the branches develop at the same time. Thus, another mechanism for branching must be invoked. Schonland\(^5\) considered the cause of the convolution and branching of discharge channels to be the space charge in front of them, but did not propose a specific mechanism.

The data of Refs. 1 and 2 indicate that the reason for leader branching is the splitting of its tip owing to flattening of its initially hemispherical shape with radius \(r_0\) because of the interaction between its charge and the residual charge of the same sign in front of the tip which accumulates as the tip moves (Fig. 3\(a\)). The flattening of the tip causes regions with radius \(r_1 < r_0\) to appear at the channel edge near the generatrices of the channel where the field strength is higher than at the center of the tip along the channel axis. Since the velocity depends on the field strength, with the passage of time the central part of the front of the tip will slow down, while the peripheral regions will accelerate. Finally, this leads to bifurcation. Let us evaluate the feasibility of this mechanism by comparing the average time interval for leader branching with the calculated time required for a residual positive charge equal to the charge in the tip to develop ahead of the tip based on the proposed mechanism and the experimental conditions. The residual charge ahead of the tip is a consequence of the corona discharge from the tip and its magnitude should be determined by the corona current \(i_k\) and the branching time \(\Delta t\). When the leader finishes developing, the potential \(\varphi \approx 2\,\text{kV}\) at its tip\(^1\) is roughly the same as the igni-