SOLIDS

Structure

Spiral Magnetic Configuration in a Thin Film with Biaxial Anisotropy


*Institute of Magnetism, Ukrainian National Academy of Sciences, Kiev, 252680 Ukraine
**Kiev Polytechnical Institute, Kiev, 252056 Ukraine

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Abstract—The conditions for the existence of a spiral magnetic configuration are determined for a thin ferromagnetic film with biaxial anisotropy. © 2000 MAIK “Nauka/Interperiodica”.

We represent the magnetization vector in terms of the angular variables in a spherical system with polar axis $x$:

$$
M = M_0(\cos \theta, \sin \theta \sin \varphi, \sin \theta \cos \varphi),
$$

where $\theta$ and $\varphi$ are polar and azimuthal angles, and $M_0$ is the saturation magnetization of the film material.

It is shown in [1–3] that in thin magnetic films the magnetization distribution can be treated as uniform over the thickness. Adopting this assumption, we shall determine one variant of a one-dimensional periodic structure whose magnetization varies in the $x$ direction.

To this end we make in Eq. (1) the substitution of variables according to Eq. (2) and integrate it with respect to $y, y', z, z'$.

As a result of these transformations the energy of the system assumes the form

$$
E = \frac{E_0}{2L_x} \left\{ \int dx \left[ \left( \frac{\partial \theta}{\partial x} \right)^2 + \alpha \sin^2 \theta \left( \frac{\partial \varphi}{\partial x} \right)^2 \right] ight.
$$

$$
- \beta \sin^2 \theta \cos^2 \varphi + (\rho + 4\pi) \cos^2 \theta
$$

$$
+ \int \int dx' dx J(x-x')(\cos \varphi(x) \cos \varphi(x')) \sin \theta(x) \sin \varphi(x') - \cos \theta(x) \cos \theta(x') \right\},
$$

where

$$
E_0 = M_0^2 L_x L_y L_z,
$$

$$
J(x) = \frac{2}{L_z} \ln \left( \frac{x^2 + L_z^2}{x^2} \right).
$$

Setting the variation of the energy functional (3) to zero we obtain an equation for the angular variables of
the magnetization field:

$$- \alpha \frac{\partial^2 \theta}{\partial x^2} + \left( \alpha \left( \frac{\partial \varphi}{\partial x} \right)^2 - \beta \cos^2 \varphi - \rho \right) \sin \theta \cos \theta$$

$$-h_z^{m}(x) \sin \theta + h_z^{(m)}(x) \cos \theta \cos \varphi = 0,$$

$$-\alpha \frac{\partial}{\partial x} \sin^2 \theta \frac{\partial \varphi}{\partial x} + \beta \sin^2 \theta \sin \varphi \cos \varphi$$

$$-h_z^{(m)}(x) \sin \theta \sin \varphi = 0.$$

In Eqs. (4) the following notation is used for determining the characteristic magnetostatic fields of the system:

$$h_z^{m}(x) = \int dx' J(x - x') \sin \theta(x') \cos \varphi(x'),$$

$$h_z^{(m)}(x) = 4\pi \cos \theta - \int dx' J(x - x') \cos \theta(x').$$

It is easy to show that the system of equations (4) can possess a particular solution that describes a helicoidal magnetic structure:

$$\theta = \pi/2, \quad \varphi(x) = kx,$$

with the following relation between the wave number $k$ of the spiral and the anisotropy constant $\beta$:

$$\beta = h(k).$$

where

$$h(k) = 4 \int_0^\infty dt \ln \left( \frac{1 + t^2}{t^2} \right) \cos(kLt)$$

$$= \frac{4\pi}{|kL|} (1 - \exp(-|kL|))$$

is the amplitude of the magnetostatic field in the film in relative units $M_0$.

Figure 1 shows schematically the distribution of the magnetic field in the film along the $x$ axis:

$$H_z^{m}(x) = M_s h(k) \cos kx$$

and the components of the magnetization of the system. It follows from the relation (7) that the solution (6) exists if the anisotropy constant satisfies the condition $\beta \leq 4\pi$.

The solution (6) of the variational equations (4) determines a helicoidal configuration corresponding to an extremum of the magnetic energy of a thin film with biaxial anisotropy. Consequently, to answer the question of the stability of the structure we shall consider a second variation of the energy near the ground state (6):

$$E_2 = \frac{E_0}{2L_x} \{ E_2(\delta \varphi) + E_2(\delta \theta) \},$$

$$E_2(\delta \varphi) = \int dx \left( \alpha \left( \frac{\partial \delta \varphi}{\partial x} \right)^2 - \beta \delta \varphi^2 \sin^2 kx \right)$$

$$+ \int dx dx' J(x - x') \delta \varphi(x') \delta \varphi(x) \sin kx \sin kx', \quad (8)$$

$$E_2(\delta \theta) = \int dx \left( \alpha \left( \frac{\partial \delta \theta}{\partial x} \right)^2 + (\rho + 4\pi - \alpha k^2) \delta \theta^2 \right)$$

$$- \int dx dx' J(x - x') \delta \varphi(x') \delta \varphi(x).$$

To eliminate ambiguity in the Fourier expansion of the variations, we shall confine the variations in the values of the wave number $q$ to the region $[-k, k]$, which is essentially the analog of the first Brillouin zone, and we shall represent $\delta \varphi$ and $\delta \theta$ in the form

$$\begin{pmatrix} \delta \varphi(x) \\ \delta \theta(x) \end{pmatrix} = \frac{1}{\sqrt{L_x}} \sum_n \sum_q \begin{pmatrix} a_q(n) \\ b_q(n) \end{pmatrix} \exp(iq + 2kn)x. \quad (9)$$

The summation in Eq. (9) extends over the admissible values of the wave numbers $q$ and the reciprocal lattice sites $n$.

On the basis of the obvious condition

$$\int_{-L_x/2}^{L_x/2} dx \exp(i(q - q' + (n - m)2k)x) = \delta_{qq'} \delta_{nm},$$

where $\delta_{nm}$ and $\delta_{qq'}$ are the Kronecker delta functions, it is easy to show that the values of the expansion coefficients have the form

$$\begin{pmatrix} a_q(n) \\ b_q(n) \end{pmatrix} = \frac{1}{\sqrt{L_x}} \int_{-L_x/2}^{L_x/2} dx \begin{pmatrix} \delta \varphi(x) \\ \delta \theta(x) \end{pmatrix} \exp(iq + 2kn)x. \quad (11)$$

Substituting the expression (9) for the variations of the magnetization field into the quadratic form (8) and performing the integration, we obtain the following result: