Electron Distribution Function in a Two-Temperature Plasma with Cold Ions

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Abstract—By modeling the dynamics of a large ensemble of particles, it is shown that slow electrons in a two-temperature plasma are in equilibrium with the electron component rather than with cold ions. The result of cooling by a cold ion component is that the number of the low-energy electrons only slightly exceeds that in the equilibrium Maxwellian distribution. © 2000 MAIK “Nauka/Interperiodica”.

1. INTRODUCTION

Due to the large ion-to-electron mass ratio, electron and ion plasma components may obey Maxwellian distributions with different temperatures. Such a system may be steady if equilibrium is sustained in each subsystem (e.g., when the electron component in a discharge is heated with an RF field, whereas the ions are in equilibrium with a neutral gas). It may also be quasi-steady (e.g., after the pulsed heating of the electron component). In the latter case, each subsystem acquires its own temperature and then the temperatures equalize. Of interest is the problem of how the electron distribution function differs from Maxwellian in the low-energy region. It is the low-energy electrons that determine the recombination flux and, thus, the ion level populations, ion line emission, and possible population inversion. Since the cross section for Coulomb scattering decreases with energy, the slow electrons undergo the most intense cooling in Coulomb collisions with cold ions. This effect influences the formation of a quasi-steady electron distribution function in a two-temperature plasma in the course of temperature equalization.

In a recent paper [1] based on the analysis of Boltzmann’s equation with the Landau collisions integral for a two-temperature plasma, it was concluded that the fraction of the low-energy electrons

\[ Y = \frac{m_z}{m_i} \zeta_n e, \]  

where \( z \) is the ion charge number and \( n_e \) is the electron density, would be in quasi-equilibrium with the cold heavy plasma component (ions or, under certain conditions, neutral atoms) and would possess its temperature. This conclusion, being very important for X-ray lasers with recombination pumping [2], requires additional verification. Computer simulations based on \textit{ab initio} principles for an ensemble of classical Coulomb particles [3, 4] can provide such verification. In this paper, we consider the equalization of electron and ion temperatures due to Coulomb collisions.

2. NUMERICAL SIMULATION OF THE DYNAMICS OF A LARGE ENSEMBLE OF PARTICLES

Let us consider a fully ionized plasma consisting of ions with mass \( m_i \) and positive charge \( +ze \) and electrons with mass \( m_e \) and charge \( -e \). The plasma is modeled by the method of molecular dynamics. We consider the time evolution of a system consisting of \( n(1 + z) \) particles housed in a cube with specularly reflecting walls. The trajectories of \( n \) ions and \( zn \) electrons are determined by solving the Newton equations

\[ \frac{d^2 r_k}{dt^2} = F_k/m_k, \quad F_k = \sum_{l \neq k}^{n(z + 1)} f_{kl}, \]

where \( r_k(t) \) is a radius-vector of the \( k \)th particle with mass \( m_k \) and charge \( q_k \). The Coulomb interaction force \( f_{kl} \) for a distance between the particles of less than \( r_0 \) was taken to be equal to the interaction force between uniformly charged interpenetrating spheres of diameter \( r_0 \) [3]. Such a modification of the Coulomb force at short distances eliminates a singularity at a zero distance and reduces the equation stiffness caused by the short-range collisions. The adopted value of \( r_0 \) is much less than the mean interparticle distance \( r_0 \ll N^{-1/3} \), where \( N \) is the particle number density; in our runs, we set \( r_0 = 0.025N^{-1/3} \). Additional runs were carried out in order to estimate the influence of the \( r_0 \) value on the characteristics a Coulomb system [4]. A particle–particle method [3] was used to solve the set of Eqs. (2). The nearest neighbors were separated out to enhance the accuracy of numerical integration.

A fully ionized plasma with \( z = 1, N_i = 10^{18} \text{ cm}^{-3} \), and initial electron and ion temperatures of \( T_e = 10 \text{ eV} \)
and $T_i = 1$ eV, respectively, was investigated. In order to embrace a greater number of slow electrons, the ion-to-electron mass ratio was chosen to be 20. According to (1), for these plasma parameters, the fraction of electrons possessing the temperature of ions ($T_i = 1$ eV) would amount to 5%. The total number of particles in the system to be modeled was taken to be $(1 + z)n = 4000$. The system dynamics was computed over the time interval $t_0 = 12.8 \times 10^{-12}$ s, which substantially exceeds the time during which a steady-state electron energy distribution is established but is less than the time of equalization of electron and ion temperatures. Over the time $t_0$, the energy exchange between the cold and hot components leads to a 3% drop in the electron temperature.

Figure 1 presents the calculated electron kinetic-energy distribution function. To make an analysis of the distribution function in the low-energy region more convenient, a logarithmic energy scale is adopted. The calculated electron distribution function differs from Maxwellian at most by 30% in that region. The time evolution of the electron kinetic energy is illustrated in Fig. 2 by the example of electrons with a low initial kinetic energy. The solid curve shows the evolution of the kinetic energy of an electron whose total energy at the initial instant is positive (“free” electron), and the dashed curve corresponds to an electron with a negative total initial energy (“bound” electron). In both cases, electrons acquire kinetic energy rather than stay in equilibrium with cold ions.

3. DISCUSSION OF THE RESULTS OF NUMERICAL SIMULATION

The results of our calculations show that the increase in the fraction of slow electrons is fairly small. Taking into account that this increase was a hundred-fold magnified by the chosen ion-to-electron mass ratio, we can expect that, for a real mass ratio, the population of ion levels will hardly be influenced by this effect. The electron distribution stated in [1] was obtained based on the spherical cavity effect (first noted by Belyaev and Budker [5]), which implies that a probe particle does not exchange energy with faster particles under the condition that the distribution function is spherically symmetric in velocity space. It is of interest to explore the spherical cavity effect, in particular, its influence on collisional relaxation in a two-temperature plasma.

An analogy with electrostatics (the Trubnikov–Rosenblut potentials) leads to a formal consequence that the slowest particle is not able to acquire any energy at all. However, it is obvious that the applicability of this result is limited, especially, in the case of slow particles. The evolution of slow electrons in Fig. 2 clearly demonstrates that the spherical cavity effect does not apply to slow particles. It was pointed out by Sivukhin ([6], p. 123) that the spherical cavity effect stems from neglecting the dependence of the Coulomb logarithm on the relative particle velocity. In deriving this effect, a low momentum transfer per collision (straight-trajectory approximation) and the binary nature of collisions were also assumed. However, a slow particle interacts simultaneously with many others.