Surface Anchoring and Temperature Variations of the Pitch in Thin Cholesteric Layers

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Abstract—The temperature variations of the cholesteric pitch in thin planar layers of cholesterics and their dependence on the surface anchoring force are investigated theoretically. It is shown that the temperature variations of the pitch in a layer are of a universal character. This is manifested in the fact that they depend not separately on the parameters of the sample but only on one dimensionless parameter $S_{d} = K_{22}dW$, where $K_{22}$ is the torsional modulus in the Frank elastic energy, $W$ is the height of the surface-anchoring potential, and $d$ is the thickness of the layer. The investigation is performed the parameter $S_{d}$ in a range where the change per unit number of cholesteric half-turns within the thickness of the layer accompanying a change in the temperature is due to the slipping of the director on the surface of the layer through the potential barrier for surface anchoring. The critical values of the parameter $S_{d}$ (which are most easily attained experimentally by varying the thickness of the layer), determining the region of applicability of the approach employed, are presented. The temperature variations of the free energy of the layer and the pitch of the cholesteric helix in the layer as well as the temperature hysteresis in the variations of the pitch with increasing and decreasing temperature are investigated for the corresponding values of $S_{d}$. Numerical calculations of the quantities mentioned above are performed using the Rapini anchoring potential. © 2000 MAIK “Nauka/Interperiodica”.

1. INTRODUCTION

Interest in detailed investigations of chiral liquid crystals (LC) has increased in recent years (see, for example, [11]). This interest is partially due to the general-physical problem (not solved thus far) of choosing the chiral order parameters characterizing chiral systems (see, for example, the review in [2]). Such investigations are also of interest because the many additional advantages of chiral LC over the nematic LC, which are ordinarily used, have not been completely realized in applications. Since cells of chiral, specifically, cholesteric, LC are now widely used as sensors of various kinds as well as for information display devices controlled by optical transparencies and for many other purposes, it is extremely important to study the properties of these LC in bounded geometries. Investigations of the optical characteristics of thin layers of chiral LC yield important information (for applications and for understanding the physics of LC) about changes in the structure of a LC in thin layers and about the dynamics of these changes.

In [3, 4] the temperature behavior of the cholesteric pitch in thin planar cholesteric layers was investigated by measuring their optical transmission spectra, which on the basis of a well-developed theory of the optical properties of chiral liquid crystals [5, 6] were interpreted in terms of the temperature variations of the parameters of the cholesteric in the layer. An unusual temperature behavior of the transmission spectra of light with wavelength of the order of the pitch of the cholesteric helix and temperature hysteresis in abrupt changes of the pitch were attributed to the deviations of the director on the surface of the sample away from the direction of alignment in the potential well of the surface-anchoring forces and the abrupt transitions of the cholesteric helix in the layer between configurations differing from one another by one half-turn of the helix in the layer. Comparing the results with the theory of temperature variations of the pitch in cholesteric layers, which was developed on the basis of the continuum theory of elasticity taking account of the surface-anchoring forces, showed that the parameters of the experimental samples were such that the mechanism of the temperature jump in the pitch did not correspond to a transition between configurations of the helix with the number of half-turns differing by one by means of the director overcoming on the surface the barrier in the surface anchoring potential. Specifically, the measured angles of deflection of the director on the surface of the layer from the direction of alignment for the temperature of the jump [3, 4] were much smaller than the critical (see below) value of this angle. Consequently, the mechanism of the jump in the pitch and the reasons why a superposition of two spectra corresponding to configurations in which the number of half-turns of the helix in the layer differs by one are present in the transmission spectra remained unknown. This requires a fur-
ther study of the question, specifically, an analysis of the situations for which a reliable theoretical description is known, especially since the pitch jumps appear in various precise investigations of layers of chiral liquid crystals, for example, in the nonlinear generation of high optical harmonics [7] and investigations with a Fabry–Perot interferometer [8]. The present paper is devoted to an analysis of one such situation, specifically, the study of the temperature variations of the cholesteric pitch in thin layers. The conditions under which the simple continuum theory of elasticity, taking account of the surface-anchoring forces, is applicable are found. For these conditions a universal description of the temperature variations of the pitch in the layer is proposed, and a theory of the temperature hysteresis of jumps in the pitch is given and experimentally observable effects are found.

2. BASIC EQUATIONS

We shall consider the temperature behavior of the pitch of a helix in a thin planar cholesteric layer, assuming the surface-anchoring forces to be identical on both surfaces of the layer and assuming the alignment axis to be the same on both surfaces. We shall use for this the expression for the free energy in the form [9]

\[ F(T) = 2W_s(\phi) + \frac{K_{22}d^2}{2}\left(\frac{2\pi}{p_s(T)} - \frac{2\pi}{p(T)}\right)^2, \tag{1} \]

where \(K_{22}\) is the torsional elastic modulus, \(W_s(\phi)\) is the surface anchoring potential, \(d\) is the thickness of the layer, \(p(T)\) is the equilibrium value of the pitch of the cholesteric helix for temperature \(T\) in a bulk cholesteric, \(p_s(T)\) is the pitch at the same temperature in a layer of thickness \(d\), and \(\phi\) is the angle of deflection of the director on the surface of the layer from the direction of alignment.

The formula (1) requires comment. The point is that the properties of a deformed cholesteric depend strongly on the ratios of the nonuniformity scale and the pitch of the helix. On scales much less than the pitch of the helix the cholesteric has the same properties as a nematic. In the opposite limit the elastic properties of the cholesteric are equivalent to those of a smectic. Consequently, the expression for the “quasi-smectic” energy (1) is valid for cells with thickness much greater than the pitch of the cholesteric helix when the surface anchoring ensures that the director is not tilted away from the normal to the cholesteric axis for the equilibrium configuration of the LC in the layer.

Since the pitch \(p_s(T)\) in the layer is uniquely related with the angle \(\phi\) and the equilibrium pitch \(p(T)\) is related with \(\phi_0(T)\)—the angle of deflection of the director on the surface of the layer away from the direction of alignment in the absence of surface anchoring, the expression for the free energy can be rewritten as a function of these angles:

\[ F(T) = 2W_s(\phi) + (2K_{22}/d)\left[\phi - \phi_0(T)\right]^2. \tag{2} \]

However, \(\phi\) and \(\phi_0(T)\) are interrelated and the relation between them can be found by minimizing the free energy (2). This gives an equation for the angle \(\phi\) as a function of temperature:

\[ \frac{\partial W_s(\phi)}{\partial \phi} + \frac{2K_{22}}{d}\left[\phi - \phi_0(T)\right] = 0. \tag{3} \]

Using Eqs. (2) and (3), the expression for the free energy can be represented in the form

\[ F(T) = 2W_s(\phi) + \left(\frac{\partial W_s(\phi)}{\partial \phi}\right)^2 \frac{d}{2K_{22}}, \tag{4} \]

where the temperature dependence of the angle \(\phi\) of deflection of the director from the direction of alignment on the surface of the layer is determined by Eq. (3). In what follows, we shall assume for simplicity (and also for making estimates) that the surface anchoring potential \(W_s(\phi)\) is determined only by one characteristic energy \(Wd\) (depth of the potential well).

We note that dividing Eqs. (2)–(4) by the depth \(Wd\) of the surface potential we find that the depth of the surface anchoring potential, the elastic modulus, and the thickness of the layer enter in these equations only through the dimensionless parameter \(S_d = K_{22}dWd\) (we note that the parameter \(S_d\) differs only by a factor from the parameter \(S = 4\pi(d/p)p_s\) used in [3, 4]). Thus, for a fixed value of \(S_d\) the temperature dependence of the orientation of the director on the surface (the angle \(\phi\)) is determined only by the form of the anchoring potential and the temperature dependence of the equilibrium pitch \(p(T)\), i.e., \(\phi_0(T)\). The temperature dependence of \(\phi\) and the free energy, which follow from the equations presented, for a fixed value of the parameter \(S_d\) will be universal, i.e., it will not depend on the number of half-turns of the cholesteric helix over the thickness of the layer (or, which is the same thing, it does not depend on the thickness of the layer).

The potential barrier between the configurations of the helix that differ by one half-turn also does not depend on the thickness of the layer and is determined only by the parameter \(S_d\). The barrier height depends on the temperature (through the function \(\phi_0(T)\)) and is determined by the expression

\[ B(T, S_d) = F(\phi, \phi_0(T), S_d) - F(\phi(T), \phi_0(T), S_d), \tag{5} \]

where \(\phi\) is a critical angle at which an abrupt change occurs in the configuration of the cholesteric helix in the layer; the equilibrium value of the angle \(\phi(T)\) is determined by the solution of Eq. (3), and the free energy \(F(\phi, \phi_0(T), S_d)\) is determined by Eq. (2). As should be the case, when the angle \(\phi(T)\) reaches the critical value the barrier height becomes zero and a transition between