Oscillatory Propagation of an Ionization–Shock Front  

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Abstract—Based on numerical simulations, we show that the oscillatory propagation of a plane-parallel ionization-shock front is possible for a typical dependence of the interstellar-medium cooling function on density and temperature. In this case, the oscillation amplitude of the shock position in the presence of an ionization front can be several times larger than its value for a single shock wave. The variations in neutral and ionized gas velocities attributable to oscillations are comparable in order of magnitude and agree with the random velocities observed in H II regions. © 2001 MAIK “Nauka/Interperiodica”.

Key words: interstellar medium, H II regions, shock waves, ionization fronts, instability

INTRODUCTION

In the literature, much attention is currently given to shock stability under conditions of strong radiative cooling of the postshock gas. The interest in this problem stems from the numerous astrophysical phenomena characterized by the propagation and interaction of radiative shock waves (supernova remnant expansion, interactions of stellar winds with circumstellar gas, and accretion flows).

A linear analysis and direct numerical calculations by Langer et al. (1981), Chevalier and Imamura (1982), Imamura et al. (1984), and Strickland and Blondin (1995) indicate that plane one-dimensional shocks can be unstable in a gas with radiative cooling if the increase in cooling rate with temperature behind the shock front is not too rapid. In this case, finite-amplitude oscillations in the shock position arise at the nonlinear stage of instability development. Such an effect also takes place in the presence of a magnetic field (Kimoto and Chernoff 1997) and when the postshock flow is spherically symmetric (Walder and Follini 1996).

However, the above results refer to the stability of single shock waves, whereas actual flows often include a set of discontinuities (shock wave, tangential discontinuity, ionization and photodissociation fronts, etc.). We therefore consider below the possibility of oscillatory propagation of a shock wave–ionization front system (ionization–shock front), which is typical of expanding H II regions. Our goal is to determine the effect of an ionization front (I-front) and the form of the cooling function on the perturbation amplitude and spectrum, as well as to estimate the possible contribution of oscillations to the observed gas velocity variations in H II regions.

STRUCTURE OF THE POSTSHOCK FLOW IN A GAS WITH RADIATIVE COOLING AND THE INSTABILITY DEVELOPMENT MECHANISM

The general pattern of variations in postshock gas parameters with allowance for radiative cooling is well known (see, e.g., the monograph by Baranov and Krasnobaev 1977): a zone of dramatic increase in density \( \rho \) and decrease in temperature \( T \) and particle velocity \( u \) at a relatively modest increase in pressure \( p \) (relaxation zone) immediately follows an adiabatic discontinuity.

Nevertheless, to determine the relaxation-zone size \( L_c \) and the distribution of gas-dynamical quantities inside this zone requires analyzing the flow properties in more detail.

Let us consider a plane one-dimensional unsteady gas motion with volume heat losses through radiative cooling. The system of equations for this motion is

\[
\frac{\partial \rho}{\partial t} + \frac{\partial \rho u}{\partial x} = 0,  \\
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + \frac{1}{\rho} \frac{\partial p}{\partial x} = 0,  \\
\frac{\partial}{\partial t} \left( \frac{1}{\gamma-1} \frac{p}{\rho} \right) + u \frac{\partial}{\partial x} \left( \frac{1}{\gamma-1} \frac{p}{\rho} \right) + \frac{\rho \partial u}{\partial x} = -\frac{\Lambda}{\rho},  \\
p = \frac{\rho k T}{m_H},
\]

where \( t \) and \( x \) are the time and spatial coordinate, respectively; \( \gamma = 5/3 \) is the adiabatic index; \( \Lambda \) is the amount of energy lost by a unit gas volume per unit time through radiative processes; \( k \) is the Boltzmann constant.
constant; $\mu$ is the mean molecular weight of gas particles; which below is taken to be unity unless specified otherwise; and $m_H$ is the hydrogen atomic mass. Next, we assume that $\Lambda = \Lambda(\rho, T)$; for typical interstellar conditions, we can also assume that $\Lambda = \rho^2\varphi(T)$ (Kaplan and Pikel’ner 1979) and that $\varphi(T)$ is proportional to the cooling efficiency (Bochkarev 1992).

Let the shock coordinate in a steady state, when $\partial/\partial t = 0$, be $x_s = 0$ and the parameters of the incoming homogeneous flow be $\rho_0, u_0, T_0$. $M_0^2 = \rho_0 u_0^2/\gamma p_0$ at $x < 0$. We denote the corresponding quantities immediately behind the adiabatic discontinuity by $\rho_s, u_s, T_s$, and $M_s$. We can then easily estimate $L_c$ from the energy equation (Strickland and Blondin 1995)

$$L_c = \frac{kT_s}{m_H \Lambda(\rho_s, T_s)}.$$  \hspace{1cm} (2)

We see from (2) that, if the characteristic cooling time $L_c/\mu$, increases with shock velocity (i.e., with $T_s$), an excess (relative to the steady state corresponding to the changed discontinuity velocity) pressure arises behind the discontinuity. Having reflected from the boundary of the region occupied by a dense cold gas, this pressure perturbation is capable of accelerating the shock front further (Strickland and Blondin 1995). Of course, this instability mechanism is efficient only in the presence of a sufficiently sharp transition zone between the heated and cold gas. The variations of $\rho, u,$ and $T$ in this zone can be easily determined by numerically integrating the system of equations (1), which reduces to a system of ordinary differential equations for steady motion. However, let us derive an approximate analytic dependence of $\rho, u,$ and $T$ on $x$, which is especially useful in selecting parameters of numerical simulations for the oscillatory propagation of a shock wave or a set of discontinuities with the relaxation zone. For this purpose, we take into account the fact that, as was mentioned above, the pressure change in the relaxation zone is not too large; and, to a first approximation, we can assume that $p = p_s = \text{const}$ at all $x > 0$. We then derive the following equation for $T(x)$ from (1):

$$\gamma L_c \frac{dT}{T_s} = \varphi(T) \frac{d(T)}{T_s},$$  \hspace{1cm} (3)

Integrating (3) yields

$$\frac{x}{L_c} = \frac{\gamma - 1}{\gamma - 1} \int_1^{T/T_s} \frac{\varphi(T)}{\varphi(T)} \left( \frac{T}{T_s} \right)^2 d\left( \frac{T}{T_s} \right).$$  \hspace{1cm} (4)

Relation (4) allows us to determine the value of $x = x_s$ at which the postshock gas temperature is equal to $T_p$, and further cooling is insignificant ($x_s$ is often taken as the exact coordinate of the relaxation-zone boundary). Specifically, for a power law $\varphi(T) \sim T^\alpha$, we find that

$$\frac{x_s}{L_c} = \frac{1}{\gamma - 1 - \alpha} \left[ 1 - \left( \frac{T_0}{T_s} \right)^{3 - \alpha} \right].$$  \hspace{1cm} (5)

Formula (5) clearly gives a slightly overestimated value of $x_s$. However, it differs from the results of numerical calculations only slightly in the $\alpha$ range $-1/2$ to $1/2$ of interest. The existence (at $x = x_s$) of a narrow zone with steep $T, \rho,$ and $u$ gradients when $M_0 \gg 1$ predicted by relation (4) is also corroborated by calculations.

**EFFECTS OF THE COOLING EFFICIENCY ON SINGLE-SHOCK OSCILLATIONS**

The nonlinear stage of perturbation development was extensively studied by Strickland and Blondin (1995) for $\varphi \sim T^\alpha$ and $0 \leq \alpha \leq 1$. Instability manifested itself at sufficiently large $M_0$ for $\alpha \leq 0.75$. Kimoto and Chernoff (1997) used a more complex form of the cooling function, including negative $\alpha$, when taking into account a magnetic field. However, their calculations were mainly applied to the dynamics of supernova remnants, so the shock velocity was $150–240$ km s$^{-1}$ and the cooling was disregarded if $T \leq 2 \times 10^4$ K. However, the shock velocity is $\sim 10–20$ km s$^{-1}$ and $T_0 \sim 10^2$ K when H II regions expand. Naturally, the temperature dependence of the cooling efficiency is also different. Therefore, before considering the motion of an ionization–shock front, we turn to our calculations of the propagation of a single shock in the circumstellar gas.

The calculations were performed by using a monotonic difference scheme of the second approximation order. An adaptive movable grid was used to increase the calculation accuracy in the vicinity of $x = x_s$. The cooling was assumed to be insignificant if $T < T_0$. However, for convenience of calculations, we assumed that the ambient temperature could not fall below $T_0$.

We performed a test calculation with parameters close to those used by Strickland and Blondin (1995): $\alpha = 0.0$ and $M_0 = 40$. At the initial instant of time, an impermeable wall was placed in the flow behind the shock at a distance $x = 0.08L_c$. Figure 1 plots $x_s(t)$ (below, $x_s$ and $t$ are given in units of $L_c$ and $L_c/\mu_{\text{ref}}$, respectively). We can see that, despite some differences in the statement of the boundary conditions, the oscillation amplitude and period satisfactorily agree with those obtained previously by different methods (Imamura et al. 1984; Wolf et al. 1989; Strickland and Blondin 1995).

As applied to the expansion of H II regions, let us consider the temperature dependence of the cooling function, which corresponds to the well-known (below denoted by $\varphi_{\text{lam}}$) cooling efficiency of the interstellar medium [see, e.g., the monographs by Kaplan and Pikel’ner (1979), Spitzer (1981), and Bochkarev (1992)].

The function $\varphi_{\text{lam}}$ is characterized by the presence of a plateau at temperatures slightly lower than $10^4$ K and by a sharp decrease with decreasing temperature at $T \leq$