\section*{Bc-Meson Decays and Lifetime within QCD Sum Rules}

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\section*{Abstract}

On the basis of three-point sum rules, the form factors for the semileptonic decays $B_c^+ \to B_s(B_s^*)l^+\nu_l$ are calculated with allowance for Coulomb corrections for the heavy quarkonium. Generalized relations associated with spin symmetry within the approach combining heavy-quark effective theory and nonrelativistic QCD are derived for form factors in the region of recoil momenta close to zero. The nonleptonic decays of the $B_c$ meson are studied on the basis of the factorization hypothesis. By summing the main exclusive modes of $c$-quark decays and by using the results of a previous analysis of $b$-quark decays, the $B_c$-meson lifetime is estimated within QCD sum rules and within nonrelativistic QCD.

\section*{1. INTRODUCTION}

In order to obtain deeper insights into the electroweak properties of heavy quarks and to measure these properties more precisely, it is desirable to have as wide a set of observations of hadrons containing heavy quarks as is possible. The doubly heavy, long-lived $B_c$ meson, which was recently observed by the CDF collaboration [1], is a new subject of such investigations.

The spectroscopy of this meson is similar to the spectroscopy of charmonium and bottomonium, because it consists of two nonrelativistic heavy quarks. It follows that nonrelativistic QCD (NRQCD) is applicable to the $B_c$ meson [2]. Available predictions for the mass spectrum of $bc$ levels were obtained in [3] within potential models and from lattice calculations. The arrangement of the excitation levels is similar to that in the excitation spectra of charmonium and bottomonium. However, the $B_c$ meson is different from them in that it does not annihilate into light quarks, gluons, or leptons. This means that all its excited states decay into lower states by emitting photons and pion pairs. Although the measured value of the $B_c$-meson mass is characterized by large uncertainties,

$$M_{B_c} = 6.40 \pm 0.39 \pm 0.13 \text{ GeV},$$

it is compatible with theoretical predictions.

The mechanism of $B_c$-meson production was studied in [4]. The simplest case is that of $e^+e^-$ annihilation, for which universal fragmentation functions describing transitions into $S$-, $P$-, and $D$-wave states were obtained by factorizing hard quark production and subsequent soft hadronization, which is reliably described by potential models. In hadron collisions, only at transverse momenta satisfying the condition $p_T \gg M_{B_c}$ is the fragmentation regime dominant; therefore, nonleading terms in $1/p_T$ (higher twists) must be included in the computational scheme at $p_T \sim M_{B_c}$. They can be evaluated on the basis of the factorization approach by taking accurately into account all $O(\alpha_s^4)$ diagrams. It turns out that nonfragmentation contributions are dominant at $p_T \sim M_{B_c}$ [4].

The measured $B_c$-meson lifetime

$$\tau[B_c] = 0.46_{-0.18}^{+0.18} \pm 0.03 \text{ ps}$$

agrees with estimates obtained both on the basis of operator-product expansion combined with NRQCD used to calculate hadronic matrix elements [5–7] and on the basis of potential quark models, where one sums the contributions of dominant exclusive modes in order to compute the total $B_c$-meson width [8, 9]. The result is

$$\tau[B_c] = 0.55 \pm 0.15 \text{ ps}.$$

On the basis of QCD sum rules, $B_c$-meson decays were studied so far in [10–13]. In [10, 11], the results obtained for the form factors were approximately one-third as great as estimates based on potential quark models, while the semileptonic and hadronic widths of the $B_c$ meson appeared to be one order of magnitude less than the widths deduced by the operator-product-expansion method. The reason behind these discrepancies was indicated in [12] and was studied in [13]: it turns out that, in the sum rules for heavy quarkonia, it is necessary to take into account $\alpha_s/\nu$ Coulomb-like corrections corresponding to the sum
of ladder diagrams, where $\alpha_s/v$ is not a small parameter, since the motion of the heavy quarks in a heavy quarkonium is nonrelativistic, $v \ll 1$. The Coulomb renormalization of the quark–quarkonium vertex increases estimates that the QCD sum rules yield for the form factors characterizing $B_d$ doubly heavy mesons (our case) and in the Lagrangian of NRQCD [2] for these, one is identical to that from [14], while the other

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supplements it. To a precision better than 10%, these relations agree with the results of sum-rule calculations, whence one can conclude that the contribution of nonleading $1/m_Q$ terms is quite small.

The semileptonic widths of the $B_c$ meson are estimated numerically on the basis of the factorization hypothesis, and the widths with respect to hadronic modes are obtained. By summing the main exclusive modes, we assess the $B_c$-meson lifetime. The result is consistent both with experimental data and with the predictions of operator-product expansion and quark models. The preferable option for the normalization scale in the nonleptonic Lagrangian for a charmed quark is discussed, and the corresponding total width of the $B_c$ meson is presented. It is indicated that the uncertainty in the values of the heavy–quark mass within QCD sum rules in a given order in $\alpha_s$ is much less than in the mass values used in the operator-product-expansion method.

The ensuing exposition is organized as follows.

In Section 2, we present a general formulation of three-point sum rules for $B_c$-meson decays with allowance for Coulomb-like corrections. Two-point sum rules for the leptonic-decay constants for mesons involving one heavy quark are analyzed in Section 3, where it is also demonstrated that three-point sum rules are convergent with respect to the threshold energy for the continuum of states of the system consisting of one heavy and one light quark. In addition, the semileptonic form factors for $B_c \to B_s^{(*)}$ decays are estimated there. On the basis of spin symmetries of heavy-quark effective theories, relations between the form factors for semileptonic decays in the soft limit at zero recoil momentum are derived in Section 4. In Section 5, we compute the partial widths of the $B_c$ meson with respect to nonleptonic decay modes and assess the lifetime of the $B_c$ meson. Section 6 is devoted to discussing the optimum estimate of heavy-hadron lifetimes. In the Conclusion, we briefly summarize the basic results of our study.

2. THREE-POINT SUM RULES
FOR THE $B_c$ MESON

In order to study the semileptonic and nonleptonic decays of the $B_c$ meson that are due to $c \to s$ transitions, we make use of the method of three-point sum rules [17]. The values of the leptonic constants for mesons in the initial and in the final state are fixed on the basis of two-point sum rules that are given by

$$\langle 0|\bar{q}_1 i \gamma_5 q_2|P(p)\rangle = \frac{f_P M_P^2}{m_1 + m_2},$$

(1)

$$\langle 0|\bar{q}_1 i \gamma_\mu q_2|V(p, \epsilon)\rangle = i \epsilon_\mu M_V f_V,$$

(2)