Mixing of Meson, Hybrid, and Glueball States*

Yu. A. Simonov

Institute of Theoretical and Experimental Physics,
Bol”shaya Cheremushkinskaya ul. 25, Moscow, 117259 Russia

Received November 28, 2000; in final form, February 7, 2001

Abstract—An effective QCD Hamiltonian is constructed with the aid of the background perturbation theory and relativistic Feynman–Schwinger path integrals for Green’s functions. The resulting spectrum displays mass gaps of about 1 GeV when an additional valence gluon is added to the bound state. The mixing of meson, hybrid, and glueball states is defined in two ways—through generalized Green’s functions and through a modified Feynman diagram technique—giving similar answers. Results for mixing matrix elements are numerically not large (around 0.1 GeV) and agree with earlier analytic estimates and lattice simulations.

1. INTRODUCTION

Gluonic fields play a double role in the dynamics of QCD. On one hand, they create a QCD string between color charges, which substantiates confinement. On the other hand, valence gluons act as color sources and can be considered as constituent pieces of hadrons on the same grounds as quarks. This is a QCD string picture of hybrids and glueballs developed analytically in [1] and confirmed by recent lattice calculations [2]. On the experimental side, the study of hybrid [3] and glueball [4] states is not yet conclusive, and more experimental and theoretical work is needed, especially a quantitative treatment of the mixing of mesons, hybrids, and glueballs.

One typical feature of the QCD string approach, supported by lattice data [2], is that the addition of each valence gluon to a hadron increases the hadron mass by 0.8–1 GeV, and this is true also for purely gluonic states—glueballs. Thus, a large mass gap exists between the meson ground state and its gluonic excitation, which makes it possible to consider gluonic admixture in a given hadron state as a perturbation. Therefore, in the zeroth approximation, one has a Hamiltonian for the diagonal states of the fixed number of quarks \( q \) and \( \bar{q} \) and valence gluons \( g \), while, in the next approximation, one calculates the mixing of states perturbatively (unless masses of the states happen to be almost degenerate, in which case one solves a matrix Hamiltonian).

The mixing of meson and hybrid states was considered previously analytically in [5, 6]. In [5], the hybrid wave function was taken in the cluster approximation, with gluon wave function of bag-model type factorized by using the \( q\bar{q} \) potential model wave function. The resulting mixing matrix elements (MME) are not large, which supports the iterative scheme described above. A similar approach, based on the nonrelativistic constituent quark and gluon model, was used in [6] to study the mixing of \( 1^{-+} \) hybrid meson and charmonium.

The mixing of glueballs and other hadrons was studied on a lattice [7], with reasonably moderate MME.

The objective of the present study is to develop a general formalism of the QCD bound states including valence gluons, with reference to the mixing of states. This formalism is based on the QCD string approach [1] and is applicable to both relativistic and nonrelativistic systems, massive or massless quarks and gluons.1)

Two features are important for this formalism. First, it is derived directly from QCD with few assumptions, supported and checked by lattice computations. Second, the only input of the theory is the fundamental string tension \( \sigma_f \), current quark masses (renormalized at the scale of 1 GeV), and strong-coupling constant \( \alpha_s \). (For the total hadron masses, one needs to subtract the self-energy for each quark and antiquark equal to 0.25 GeV, but this does not affect wave functions and mixings, which will be the main goal of this study.)

The plan of the article is as follows. In the next section, we develop the background perturbation theory to separate valence gluons from the background

---

1) A short version of an approach in the same direction with a different form of matrix elements appeared recently in [8]. The results obtained in [8] are similar to ours qualitatively, with some numerical distinctions.
In (3), the arbitrary weight of averaging over the gluon propagating in the background is identified and the MME is written in terms of solutions of the diagonal Hamiltonian by using the formalism of modified plane waves.

In Section 3, the part of the Hamiltonian responsible for the mixing of mesons, hybrids, and glueballs is treated in the form of a perturbation series in $g^2$ with calculations within other models, lattice simulations, and experimental data.

In Section 4, concrete calculations of MME are presented for the meson–hybrid case based on the Green’s function formalism. An analogous treatment of the meson–glueball case is given in Section 5.

In the concluding section, a discussion is given of the results obtained in this study and a comparison is drawn with calculations within other models, lattice simulations, and experimental data.

2. GREEN’S FUNCTIONS AND HAMILTONIAN

In this section, we follow the procedure developed in detail in [9, 10] (for earlier references, see [11]); therefore, we recapitulate here only the main points. The total gluonic field $A_\mu$ is represented as the sum

$$A_\mu = B_\mu + a_\mu,$$

(1)

where $B_\mu$ is the nonperturbative part and $a_\mu$ is the perturbative (or valence gluon) part, which will be treated in the form of a perturbation series in $ga_\mu$. One can formally avoid the problem of double counting using the 't Hooft identity [9], which allows one to represent the partition function as

$$Z = \frac{1}{N^N} \int e^{-S(A)}DqD\bar{q}DA$$

(2)

$$= \frac{1}{N} \int \eta(B)e^{-S(B+a)}DBDaDqD\bar{q},$$

where $N$ and $N'$ are normalization constants.

Here, $S$ is total Euclidean action and $\eta(B)$ is an arbitrary weight of averaging over $DB$. Expanding $S(B + a)$, one obtains

$$S(B + a) = S(B) + S_1(a, B) + S_2(a, B) + S_3(a, B) + S_4(a) + S_1(a, q, \bar{q}).$$

(3)

In (3), $S_n(a, B)$ denotes terms of power $n$ in $a_\mu$, and $S_1(a, q, \bar{q})$ is the mixing term, which will be of main interest in Section 3. The terms $S_2$ and $S_4$ contain additional powers of $g$ and can be treated perturbatively; the terms $S_1(a, B)$ was considered in [10] and was shown to yield a small correction to the leading terms $S(B)$ and $S_2(a, B)$, the latter defining the valence gluon propagating in the background $B_\mu$ with the Green’s function (in the background Feynman gauge [9–11])

$$G^{(g)}_{\mu\nu} = (D^2\delta_{\mu\nu} + 2igF_{\mu\nu})^{-1},$$

(4)

$$D_\lambda = \partial_\lambda - igA_\lambda,$$

$$F_{\mu\nu} = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + gf^{abc}A_\mu^bA_\nu^c.$$  

The explicit form of all terms in (3) and of the corresponding Green’s functions is given in [9, 10]; here, we only quote the results for the Green’s function of the state containing a quark, an antiquark, and any number of valence gluons in the leading $N_c$ approximation:

$$G_{\bar{q}q(ng)}(X, Y) \sim \langle \Gamma_i S_q(x, y) \prod_{i=1}^n G^{(g)}_{\mu\nu}(x, y)S_q(x, y)\Gamma_f \rangle_B.$$  

(5)

For (multi)glueball Green’s function, one has a similar representation with the missing factors $S_q$ and $S_{\bar{q}}$. Here, $\Gamma_i$ and $\Gamma_f$ are initial and final vertex operators and $\langle ... \rangle_B$ means averaging over fields $B_\mu$ with the weight $\eta(B)$; the quark Green’s function $S_q$ can be written as

$$S_q = -i(\hat{D} + m)^{-1} \eta(B)$$

(6)

$$= -i(m - \hat{D})(m^2 - \hat{D}^2)^{-1} = -i(m - \hat{D})(m^2 - \hat{D}^2 - \sigma_{\mu\nu}F_{\mu\nu})^{-1}.$$  

In what follows, we will treat spin-dependent terms for the gluon and quark Green’s functions in (4) and (6) as a small correction—this is supported by exact calculations and by a comparison with experimental and lattice data [1]. Accordingly, one can use, for the parts quadratic in $D_\mu$, the Feynman–Schwinger (world-line) path-integral representation (see [12, 13] and references therein)

$$\frac{(m^2 - D_\mu^2)}{xy}^{-1} \prod_0^\infty \int ds \int (Dz)_{xy} e^{-K} \exp(i\int_B^x \mu d\tau),$$

(7)

$$K = \frac{1}{4} \int_0^s \left(\frac{dz_\mu}{d\tau}\right)^2 d\tau.$$  

It is important that background field $B_\mu$ enters into (7) only in the exponentials, and one can show [12] that, in the total Green’s function (5), all these exponentials combine, in the leading $N_c$ approximation, into products of Wilson loops. Here, we additionally make the assumption, supported by lattice data, that Wilson loop has a minimal-area law behavior with the string tension $\sigma_f$ (for mesons and hybrids) and set $\sigma_{adj} = 9/4\sigma_f$ for $2g$ glueballs. Thus, one reduces the system of $q, \bar{q}$, and $n$ gluons to an open string