Theory of Nutation of a Nonrigid Earth

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Abstract—A general theory of terrestrial nutation is proposed assuming that the Earth is made up of four envelopes (atmosphere, mantle, fluid core, and solid core) and taking account of all important forces (viscous, electromagnetic, etc.). A theory for the effect produced on the Earth’s nutation by viscous forces in the fluid core is developed based on experimental data on the viscosity of molten iron under pressure. The proposed theory predicts nutation in longitude and inclination with an rms deviation of 0.35 milliarcseconds. © 2001 MAIK “Nauka/Interperiodica”.

1. INTRODUCTION

The axis of the Earth’s figure, instantaneous rotational axis, and angular-momentum axis move with respect to an inertial reference frame due to the gravitational forces of the Sun, Moon, and planets. This is called precessional–nutation motion. The precessional period is \( \sim 26,000 \) yrs. Over the time during which telescope observations have been available, \( \sim 300 \) yr, we can assume that the precession forms a linear (or secular) motion of the axis with the nutation harmonics superimposed on it. The orbits of the Earth and Moon, as well as those of other planets revolving around the Sun, determine the periods of the nutational harmonics. The period of the main harmonic is 18.6 yr and is related to the period of motion of the nodes of the Moon’s orbit. The maximum amplitude of the nutation is \( \sim 9'' \). The remaining nutational harmonics have smaller amplitudes.

The Earth’s nutation is an elliptical motion, since it is the sum of two circular motions with equal periods but different amplitudes and opposite directions. We shall call counterclockwise and clockwise motions observed from the north celestial pole “forward” and “reverse” motions, respectively. Another representation of the nutation is a decomposition into two components: nutation in longitude and in inclination. The relationship between these two representations is discussed in the appendix. We shall take the periods (or frequencies) of the nutational harmonics for reverse motions to be negative and those of forward motions to be positive.

The traditional method for constructing a theory of the Earth’s nutation is the following. An amplitude–frequency transformation function is calculated for a given model of the Earth, and this function is multiplied by the amplitudes of nutational harmonics computed for a completely rigid Earth. Then, corrections are added taking into account the difference between the model and real Earth. Using this method, Wahr [1, 2] obtained nutational series composed of 106 harmonics constructed for an elastic, ellipsoidal Earth with a fluid outer core and elastic mantle. In 1980, this series was accepted by the International Astronomy Union (IAU) as a standard for astronomical computations [3]. The model determines the normal modes (i.e, resonance properties) of the Earth. The model employed by Wahr [1, 2] defines three normal modes: the Chandler wobble (CW), nearly diurnal nutation (FCN), and a tilted mode (TOM). The difference between the model and real Earth leads to changes in the frequencies of the normal modes (and to the appearance of new modes), as well as to changes in the amplification. This is especially important when the frequencies of the lunar–solar potential are close to the frequencies of normal modes.

Beginning in the early 1980s, regular radio observations using very long baseline interferometry (VLBI) were begun. At present, VLBI observations make the most important contribution to the determination of nutational motions [4]. For most harmonics, the accuracy of VLBI measurements of nutation amplitudes is no less than 0.1 milliarcsecond (mas). The results of 18 years of VLBI observations have introduced some corrections to the IAU1980 theory [4]. The differences between the observations and theory are within \( \pm 15 \) and \( \pm 5 \) mas for nutation in longitude and inclination, respectively. These differences appreciably exceed the observational errors, thus verifying the reliability of these results.

Improvements in the accuracy of observations have shown that Wahr’s [1, 2] nutation theory must be replaced by an improved theory. In 1994, the XXII IAU General Assembly (The Hague, Netherlands) created a working group on the “Nonrigid Earth
Nutation Theory.” The main goal of this working group was to develop a new nutation theory that could be used to determine the Earth’s position in space with an accuracy to better than one milliarcsecond. The report of the working group [5] considers, in detail, the problems that must be solved to construct a new theory. The working group included effects produced on the nutation by the fluid core, oceans, and atmosphere among the most poorly modeled phenomena. We accordingly focus on these effects.

The basis of our theory is the analytical approach of [6], which can be outlined as follows. We first write a system of equations for the angular momenta of the whole Earth and the envelopes included in the model (in [7], a fluid core and solid core), except for the mantle. We obtain a system of algebraic equations in the frequency domain. The solution of the homogeneous system yields the frequencies and amplitudes of the normal modes. The solution of the nonhomogeneous system is represented as the product of the transformation function and the nutation amplitudes for a completely rigid Earth. We then add corrections taking into account additional effects, such as inelastic dissipation in the mantle and the effect of the oceans. The unknown internal parameters of the Earth (such as the compression of the core–mantle boundary) are determined through fitting to obtain the best agreement between theory and observation.

All Earth-nutation theories can be divided into two groups: empirical theories and theories based on solutions of equations of rotational motion. Empirical theories derive the coefficients of the transformation function through fitting to obtain the best agreement between theory and observation. The SF2000 theory [7] and the theory of Herring [8] are examples of this type of theory. Theories based on solutions of rotational equations are, in turn, divided into numerical, analytical, and semi-analytical categories. Numerical theories are based on numerical integration of equations and analytical theories on analytical solutions of the equations of rotational motion, with the Earth’s internal parameters specified by some model for the Earth’s internal structure [10]. However, these models cannot completely determine all of the internal parameters of the Earth with sufficient accuracy. Therefore, semi-analytical theories, which estimate unknown internal parameters based on the best agreement between theoretical nutation amplitudes and observations [6, 10], are most commonly used. Our theory is semi-analytical.

The main distinctions of our theory from previous semi-analytical theories are the following:

1) the atmosphere is included in the momentum equations simultaneously with the solid and fluid cores, yielding a six-dimensional system of equations for the complex amplitudes;
2) the viscosity of the fluid core is taken into account, which, together with account of the magnetic field, leads to a magneto-viscosity tensor;
3) atmospheric tides are taken into account.

Section 2 presents five different models for the Earth for which we have computed nutational harmonics, while Section 3 describes the method used to take into account the atmosphere, viscosity, and magnetic field. Section 4 considers the results of calculations of the nutational harmonics for the given models, and Section 5 discusses our results.

2. MODELS FOR THE EARTH

Each model assumes that the Earth rotates and is ellipsoidally stratified.

Model A includes the atmosphere, mantle, fluid core, and solid core. This model takes into account torque due to viscous forces and neglects that due to magnetic forces. We adopt the corrections to the inelasticity of the mantle and the oceans, as well as the rigid-body nutation amplitudes, from [11]. The model fits the compression of the core–mantle boundary in order to match the real part of the calculated amplitude of the annual reverse nutation with the observed amplitude. The viscosity is varied to obtain the best agreement with the observations. The remaining parameters are taken from the PREM model [12] (presented in [11]), except for the dynamical compression of the whole Earth, which is taken to be \( e = 0.003284915 \) [11]. We adopt the atmospheric parameters from [13, 14].

Model B is the same as Model A except for the elimination of the atmosphere as one of the Earth’s envelopes.

Model C is the same as Model A except for the elimination of the solid core. The fluid core occupies the entire space inside the mantle.

Model D is the most complete. The Earth consists of the atmosphere, mantle, fluid core, and solid core. The Earth’s core is modeled taking into account electromagnetic and viscous forces. In the simplest case, the geophysical dynamical compression of the whole Earth is taken from [6] and is \( e = \frac{C - A}{A} = 3.284520155008 \times 10^{-3} \). To fit the theoretical nutation amplitudes to the observed amplitudes, we vary the independent components of the magneto-viscosity matrix \( S_{ab} \) and the elasticity parameters \( \gamma \) and \( \kappa \). The nutation amplitudes are calculated for various nutation series for the rigid Earth, taking into account corrections for dissipation in the mantle (calculated for various dissipation models) and for the oceans.