Production of $P$-Wave Charmonium States in Two-Particle Decays of $B_c$ Mesons

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Abstract—On the basis of the model of hard one-gluon exchange, the two-particle hadronic decays of $B_c$ mesons into $S$- and $P$-wave charmonium states, $B_c \rightarrow X_{cc}\pi(\rho)$, are considered at high momentum transfers and in the nonrelativistic approximation. It is shown that the width with respect to $B_c$-meson decay into $S$-wave charmonium states is two times greater than the width with respect to $B_c$-meson decay into $P$-wave states and that the yield of $J/\psi$ mesons in the cascade processes of $B_c$-meson decay via the formation and radiative decay of $P$-wave charmonium states is approximately 8% of the yield of directly produced $J/\psi$ mesons. © 2001 MAIK “Nauka/Interperiodica”.

1. INTRODUCTION

Information about $B_c$ mesons can be obtained most directly from their decays producing $J/\psi$ mesons in the final state [1]. At present, $B_c$ mesons are sought in two main decay channels,

$$B_c \rightarrow J/\psi + l + \nu_l$$

(1)

and

$$B_c \rightarrow J/\psi + \pi(\rho).$$

(2)

The first observation of the $B_c$ meson by the CDF collaboration at Tevatron [2] was based on an analysis of the semilepton decay mode (1). Although the width with respect to the decay process (2) is less than the width with respect to the decay process (1), two-particle hadronic decays may appear to be more promising from the point of view of isolating a signal from $B_c$-meson production and of determining the $B_c$-meson mass. First, all final-state particles in decays (2) are detectable hadrons, and this makes it possible to determine precisely the features of the $B_c$ meson. Second, the momentum transfer to the spectator quark is much greater in decays (2) than in decays (1). Allowance for this effect considerably increases the theoretical estimate of the $B_c \rightarrow J/\psi\pi(\rho)$ decay width in relation to that calculated within the spectator model, where the decay width is controlled by the overlap integral of the nonrelativistic wave functions of the initial- and final-state quarkonium. In [3] and, more recently, in [4], it was found that, within the model of hard one-gluon exchange, one can expect a nearly fourfold increase in the theoretical estimate of the $B_c \rightarrow J/\psi\pi(\rho)$ decay width. In the present study, we consider the following two-particle hadronic decay of the $B_c$ meson into the $S$- and $P$-wave states of charmonium ($X_{cc}$):

$$B_c \rightarrow X_{cc} + \pi(\rho).$$

(3)

It will be shown that the width with respect to two-particle hadronic $B_c$-meson decay producing a $J/\psi$ meson in the final state increases further by about 8% upon taking into account the contribution of cascade processes leading to $J/\psi$-meson production via the radiative decays of $P$-wave charmonium ($X_{cc}$) states generated in $B_c$-meson decays along with either a $\pi$ or a $\rho$ meson.

2. HARD-EXCHANGE MODEL

In the nonrelativistic approximation, we disregard the binding energy of quarks and antiquarks in the $B_c$ meson and in the charmonium $X_{cc}$. The $B_c$-meson mass is then equal to the sum of the $b$- and the $c$-quark mass, $m_1 = m_b + m_c$; accordingly, the charmonium mass is $m_2 = 2m_c$. Furthermore, the quark and the antiquark in the $B_c$ meson or in $X_{cc}$ move at the same 4-velocities,

$$v_1 = \frac{p_1}{m_1} = \frac{p_b}{m_b} = \frac{p_c}{m_c};$$

(4)

$$v_2 = \frac{p_2}{m_2} = \frac{p_c}{m_c} = \frac{p'_c}{m_c}. $$

(5)

In order to secure this condition for the quark and the antiquark in $X_{cc}$ after weak $b$ decay [$b \rightarrow c\pi(\rho)$], the 4-momentum transfer $k$ to the spectator $c$ quark must be quite high, with the virtuality being

$$|k^2| = \frac{m_2^2}{4m_1}(m_1^2 - m_2^2 - m_3^2) \gg \Lambda_{QCD}^2.$$

(6)
where \( m_3 \) stands for the \( \pi^- \) or the \( \rho^- \) meson mass. For various \( S^- \) and \( P^- \) wave \( X_{cc} \) states of mass \( m_2 = 3.0 - 3.5 \text{ GeV} \), \( |k^2| \) lies in the range 1.0–1.2 \( \text{GeV}^2 \). So high a value of the momentum transfer violates the condition that ensures the applicability of the spectator model and which requires a significant overlap of the wave functions for the initial- and the final-state heavy meson.

Here, use is made of the general covariant formalism proposed in [5] for calculating the cross sections for the production of \( S^- \) and \( P^- \) wave heavy quarkonia and their decay widths at high momentum transfers between the quarks involved. In the leading nonrelativistic approximation in the relative momentum of the quark and the antiquark in quarkonia, the expression for the amplitude of the decay of the \( (cc) \) bound state into the \( (\bar{c}c) \) bound state in this approach has the form

\[
A(p_1, p_2) = \int \frac{d\rho_1}{(2\pi)^3} \sum L_{1z} S_{1z} \Psi_{L_1,S_{1z}}(\rho_1) \times \langle L_1 L_{1z}; S_{1z} J_{1z} \rangle \int \frac{d\rho_2}{(2\pi)^3} \sum L_{2z} S_{2z} \Psi_{L_2,S_{2z}}(\rho_2)
\]

\[
\times \langle L_2 L_{2z}; S_{2z} J_{2z} \rangle \mathcal{M}(p_1, p_2, q_1, q_2),
\]

where \( p_1 (p_2), J_1 (J_2), L_1 (L_2), \) and \( S_1 (S_2) \) are, respectively, the 4-momentum, the total angular momentum, the total orbital angular momentum, and the total spin of the initial (final) bound state; \( \mathcal{M}(p_1, p_2, q_1, q_2) \) is the quantity that is obtained from the hard amplitude of the decay process (3) by cutting off the fermion lines of the initial and the final meson (see figure); \( \Psi_{L_{1z}S_{1z}}(\rho) \) stands for the nonrelativistic wave functions of the initial and the final meson.

We now introduce the operators \( \Gamma_{SS,}(p, q) \) that project the state of a free quark–antiquark pair onto the bound state with fixed quantum numbers. To first-order terms in the relative momentum \( q \) of the quark and the antiquark, they can be represented as

\[
\Gamma_{S_1S_{1z}}(p, q) = \sqrt{m_1} \left( \frac{m_c}{m_1} \hat{p}_1 - \hat{q}_1 + m_c \right) \hat{A}_1
\]

\[
\times \left( \frac{m_b}{m_1} \hat{p}_1 + \hat{q}_1 - m_b \right),
\]

where \( \hat{A}_1 = \gamma_5 \) for \( S_1 = 0 \) and \( \hat{A}_1 = \hat{\epsilon}(S_{1z}) \) for \( S_1 = 1 \), and as

\[
\Gamma_{S_2S_{2z}}(p, q) = \sqrt{m_2} \left( \frac{m_c}{m_2} \hat{p}_2 + \hat{q}_2 - m_c \right) \hat{A}_2
\]

\[
\times \left( \frac{m_b}{m_2} \hat{p}_2 - \hat{q}_2 + m_b \right),
\]

where \( \hat{A}_2 = \gamma_5 \) for \( S_2 = 0 \) and \( \hat{A}_2 = \hat{\epsilon}(S_{2z}) \) for \( S_2 = 1 \); here, \( \hat{\epsilon}(S_{1z,2z}) \) is the polarization vector of the spin-1 particle. It should be noted that the color factor \( \delta^{ij}/\sqrt{3} \), which takes into account the fact that the quark and the antiquark in the meson are in the color-singlet state, is omitted in the above expressions for the projection operators \( \Gamma_{SS,}(p, q) \).

Using the projection operators (8) and (9), we can represent the amplitude \( \mathcal{M}(p_1, p_2, q_1, q_2) \) in the form

\[
\mathcal{M}(p_1, p_2, q_1, q_2) = \text{tr} \left[ \Gamma_{S_1S_{1z}}(p_2, q_2) \gamma^\beta \Gamma_{S_1S_{1z}}(p_1, q_1) \mathcal{O}_\beta \right],
\]

where, in the case of a \( \pi^- \) meson in the final state, we have

\[
\mathcal{O}_\beta = \mathcal{O}_\beta^1 + \mathcal{O}_\beta^2,
\]

\[
\mathcal{O}_\beta^1 = G_F \frac{16}{\sqrt{3}} \pi \alpha_s \pi V_{bc} f_\pi a_1 \hat{p}_3
\]

\[
\times (1 - \gamma_5) \left( \frac{-\hat{x}_1 + m_c}{x_1 - m_c^2} \right) \frac{\gamma_\beta}{k^2}
\]

\[
\mathcal{O}_\beta^2 = G_F \frac{16}{\sqrt{3}} \pi \alpha_s \pi V_{bc} f_\pi a_1 \frac{\gamma_\beta}{k^2}
\]

\[
\times \left( \frac{-\hat{x}_2 + m_b}{x_2 - m_b^2} \right) \hat{p}_3 (1 - \gamma_5).
\]

Here,

\[
\hat{x}_1 = \frac{m_b}{m_1} \hat{p}_1 + \hat{q}_1 - \hat{p}_3,
\]

\[
\hat{x}_2 = \frac{m_c}{m_2} \hat{p}_2 + \hat{q}_2 + \hat{p}_3,
\]

\[
\hat{k} = \frac{m_c}{m_1} \hat{p}_1 - \frac{m_c}{m_2} \hat{q}_1 + \hat{q}_2,
\]

\( p_3 \) is the momentum of the \( \pi^- \) meson, \( f_\pi \) is its leptonic-decay constant, the factor \( a_1 \) takes into account hard gluon corrections to the effective four-fermion vertex \([6]\), \( \alpha_s \) is the strong-interaction constant, \( G_F \) is the