The Effect of Diffraction-Induced Friction

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Abstract—The effect of radiative friction emerging during the transverse motion of a screen with a slit relative to the luminous flux is considered. It is shown that the effect is associated with the diffraction of light. Although this effect is weak, it must be taken into consideration in high-precision measurements like those for detecting gravity waves. The effect is analyzed for various types of slits and in the framework of various diffraction modes. © 2002 MAIK “Nauka/Interperiodica”.

1. INTRODUCTION

During the last several decades, considerable advances were made in the theory of physical measurements. The methods developed in this field have made it possible to study new physical phenomena mainly related to quantum physics and to carry out experiments (such as the detection of gravity waves) which were formerly impossible. The following example illustrates the complexity of such a problem: the detection of gravity waves involves the measurement of the displacement of one of the mirrors by a distance on the order of $10^{-16}$ cm [1]. In high-precision measurements, the effects which could be formerly disregarded due to their smallness must be taken into account. One of such effects is diffraction-induced friction, which was introduced during an analysis of the Heisenberg microscope [2], where it limits considerably the sensitivity of the measuring circuit. The essence of this effect can be described as follows. Let a photon beam be incident on a moving screen with a slit. As a result of diffraction from the slit and the Doppler effect, photons acquire an additional momentum transferred in the direction of motion of the slit. In accordance with the momentum conservation principle, an equal and opposite momentum is transferred to the screen, resulting in friction. This effect is close to the Robertson–Poynting light friction [3, 4] observed for an astronomical object as a consequence of the Maxwell electromagnetic theory.

In [2], only the main principles of this effect were formulated. Subsequent discussions proved that a more rigorous analysis of this effect is required. This article is devoted to a detailed description of this effect. In Section 2, the basic equation for the diffraction-induced frictional force is derived. An analysis of the effect for various types of slits is carried out in Section 3 for two models of diffraction, which provide different descriptions of the radiation intensity for large diffraction angles.

2. BASIC EQUATION

We choose a laboratory frame of reference which is fixed relative to an observer (the quantities related to this frame are primed). As a model, we take an ideal conductor in the form of a flat thin screen lying in the plane $z' = 0$ with a narrow long slit cut in it along the $y'$ axis. The slit width amounts to several wavelengths of the incident light (see figure). In such a formulation of the problem, diffraction effects are independent of $y'$ and the light intensity is reduced to the solution of a 2D diffraction problem. A plane phonon beam incident along the normal to the screen will be defined by frequency $\omega_0'$. In order to register the momentum, we mentally arrange a series of photodetectors around the slit for measuring the energy of photons incident on them. It is well known that the momentum and energy of photons are connected through the relation $p = E/c$ ($c$ is the velocity of light). Consequently, the total momentum transferred to the screen by the luminous flux is given by

$$dp_x = -\frac{1}{c}\int \sin \Theta dE,$$  \hspace{1cm} (1)

where $\Theta$ is the smallest angle formed with the normal to the screen and the integral is taken over an arbitrary surface bounding the slit. The energy $dE$ of the luminous flux hitting a detector during the time interval $dt$ in the solid angle $d\Omega$ visible from the side of the slit is determined by the intensity of the light wave passing through the slit, $I(\Theta, \omega) = I(\Theta)\delta(\omega - \omega_0)$, through the relation

$$dE = I(\Theta, \omega)S\cos \Theta dt d\omega d\Omega.$$  \hspace{1cm} (2)

Here, $S$ is the area of the slit and $S\cos \Theta$ is the area of the slit visible from the detector side. It is convenient to
carry out a further analysis for the case when the detectors are arranged at a large distance behind the slit so that the slit can be regarded as a point source with a given angular distribution of intensities \( I(\Theta) \). In this approximation, light waves from light sources distributed over the surface of the slit propagate in parallel to an arbitrarily chosen detector (as in the Huygens–Fresnel diffraction model).

Let us now assume that the slit moves at a velocity \( V \ll c \). In this approximation, we will omit the terms of the order of \( V^2/c^2 \) in subsequent calculations. We seek the force acting on the screen and proportional to \( V \). This simplifies the calculations since the terms containing \( \sqrt{1-v^2/c^2} \), as well as the relativistic change in the width of the slit and the detectors, are disregarded in the transformations of the wave vector upon a transition to another frame of reference.

In the laboratory frame of reference, a diffraction pattern varying with time is formed around the slit; for this reason, it is convenient to pass to a reference frame in which the slit is stationary. In this frame of reference, a wave is incident on the screen at a small angle \( \beta = V/c \) and the field distribution will be characterized by the intensity \( I(\Theta, \beta) \). The force acting on the screen is given by

\[
F_x = \frac{dp_x}{dt} = -K \int I(\Theta, \beta) \cos(\Theta + \beta) d\Theta. \tag{3}
\]

Here, \( K = S\Delta \phi/c \), \( \Delta \phi \) being the angular size of the detectors along the \( y \) axis. The second term in the parentheses in the integrand is the initial value of the photon momentum which would be transferred to an absorbing screen without a slit:

\[
F_x^{\text{initio}} = -\frac{\beta W}{c},
\]

where

\[
W = K \int I(\Theta, \beta) \cos \Theta d\Theta
\]

is the power of diffracted light. Photons that do not participate in diffraction create no frictional force; for this reason, the effect is referred to as diffraction-induced friction. Let us consider the change in formula (3) upon a transition to the laboratory frame of reference.

In the laboratory frame, the photon frequency (energy) varies in different angular directions in accordance with the Doppler effect: \( d\omega = d\Theta (1 + \beta \sin \Theta) \). Consequently, the force acting on the screen can be expressed through the angular distribution of intensity in a reference frame in which the slit is stationary as follows:

\[
F_x' = -K \int I(\Theta, \beta) (\sin \Theta + \beta + \beta \sin^2 \Theta) \cos \Theta d\Theta. \tag{4}
\]

This expression includes only the force created by the wave passing through the slit and should be supplemented with the force created by the wave reflected from the surface of the screen (we consider the case of ideal reflection). According to [5], if we consider diffraction as radiation emitted from the screen edges, its intensity turns out to be the same in the half-planes in front of and behind the screen; consequently, the value of friction must be doubled:

\[
F_x^{\text{full}} = 2F_x'.
\]

We will now analyze the coefficient of diffraction-induced friction (\( m \) is the mass of the screen),

\[
\sigma = -\frac{F_x^{\text{full}}}{mV} = \frac{W}{mc^2} \delta,
\]

deexpressed in terms of an auxiliary coefficient \( \delta \) which depends only on the intensity distribution of the diffraction pattern:

\[
\delta = 2 \frac{\pi/2}{\pi/2} \int_{-\pi/2}^{\pi/2} \{ I(\Theta, \beta)(1 + \sin^2 \Theta) + I_\beta(\Theta, \beta) \sin \Theta \} \beta = 0 \cos \Theta d\Theta
\]

\[
\int_{-\pi/2}^{\pi/2} I(\Theta) \cos \Theta d\Theta.
\]