The necessity of considering transverse shears in the problems of beam bending was first recognized by Timoshenko [1]. On the basis of the virtual work principle, the Timoshenko’s theory was extended in [2] to the case of isotropic plates. In order to take into account a nonuniformity in the distribution of transverse shears over cross sections of a shell, the shear correction factor $k$ is introduced in this theory. To date, this factor is usually taken to be equal to $k = \frac{5}{6}$ or $\frac{\pi^2}{12}$ [3]. In [4], it was shown that a value of $k = 1$ is more preferable when the procedure of recovering the transverse components of the stress tensor by integrating the equations of the three-dimensional elasticity theory is used in the linear theory of Timoshenko shells in the approach proposed in [2]. It was this choice that made it possible to construct the mathematically consistent and geometrically noncontradictory linear theory of Timoshenko shells. In this study, the results obtained in [4] are extended to anisotropic shells with finite deflection.

1. Let us consider a thin anisotropic shell of a constant thickness $h$. We assume that an elastic-symmetry surface parallel to the face surfaces $S^-$ and $S^+$ exists at each point of the shell. As an initial surface $S$, we take a shell surface spaced from the face surfaces by $\delta^-$ and $\delta^+$; i.e., $h = \delta^+ - \delta^-$. Let the initial surface be associated with the orthogonal curvilinear coordinates $\alpha_1$ and $\alpha_2$ measured along the lines of principal curvatures. The coordinate $\alpha_3$ is measured in the direction of increasing the outer normal to the surface $S$ (see figure).

The equilibrium equations of the elasticity theory for a thin shell, which has a finite deflection and whose face-surface metrics can be identified with the metric of the initial surface, have the form [5]

$$\begin{align*}
1 \frac{\partial \sigma_{ij}}{A_1 \partial \alpha_i} + 1 \frac{\partial \sigma_{ij}}{A_j \partial \alpha_j} + & B_i (\sigma_{ii} - \sigma_{jj}) \\
+ & 2B_j \sigma_{ij} + k_j \Sigma_{i3} = 0 \quad (i \neq j), \\
1 \frac{\partial \Sigma_{13}}{A_1 \partial \alpha_1} + 1 \frac{\partial \Sigma_{23}}{A_2 \partial \alpha_2} + & \frac{\partial \sigma_{33}}{\partial \alpha_3} + B_1 \Sigma_{13} \\
+ & B_2 \Sigma_{23} - k_1 \sigma_{11} - k_2 \sigma_{22} = 0, \\
\Sigma_{i3} = & \sigma_{i3} + \Theta_1 \sigma_{1i} + \Theta_2 \sigma_{2i}, \\
\Theta_i = & \frac{1}{A_i} \frac{\partial u_i}{\partial \alpha_i} - k_i u_i, \quad B_i = \frac{1}{A_i A_j} \frac{\partial A_j}{\partial \alpha_j},
\end{align*}$$

where $u_i$ are the displacements of the shell points, $\sigma_{ij}$ are the stresses, $A_1$ are the Lamé constants, and $k_i$ are the curvatures of the coordinate lines. Hereafter, $i, j = 1, 2$ and $\alpha, \beta = 1, 2, 3$. 

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The relationships of the generalized Hooke’s law can be written in the form [4]

$$\sigma_{ij} = \sum_{l \leq m} b_{ijlm} e_{lm}, \quad \sigma_{i3} = \sum_{l} b_{i3lm} e_{i3},$$

$$i, j, l, m = 1, 2.$$  

When constructing our theory, we used the modified Timoshenko hypothesis [4, 6] on the linear distribution of tangential displacements over the shell thickness:

$$u_i = N^{+}(\alpha_3)\nu^+_i + N^{-}(\alpha_3)\nu^-_i, \quad u_3 = \nu_3,$$

$$N^{+}(\alpha_3) = \frac{\delta^+ - \alpha_3}{h}, \quad N^{-}(\alpha_3) = \frac{\alpha_3 - \delta^-}{h},$$

where $$\nu^+_i(\alpha_1, \alpha_2)$$ and $$\nu^-_i(\alpha_1, \alpha_2)$$ are the tangential displacements of the face surfaces $$S^+$$ and $$S^-$$, respectively, uniformly and linearly distributed over the shell thickness [7].

Introducing displacements (3) into the stress–strain relationship of the nonlinear theory of elasticity [5] for a thin finite-deformation shell and assuming that the transverse shear strains and tangential strains are distributed, respectively, uniformly and linearly over the shell thickness, we obtain

$$\varepsilon_{ij} = N^{+}(\alpha_3)E_{ij}^+ + N^{-}(\alpha_3)E_{ij}^-, \quad \varepsilon_{i3} = E_{i3}^+, \quad \varepsilon_{33} = 0,$$

$$E_{ij}^+ = \frac{1}{A_1} \frac{\partial \nu^+_i}{\partial \alpha_i} + B_j \nu^+_j + k_i \nu^-_i + \frac{1}{2} (\theta^+_i)^2 (i \neq j),$$

$$E_{12}^+ = \frac{1}{A_1} \frac{\partial \nu^+_1}{\partial \alpha_1} + \frac{1}{A_2} \frac{\partial \nu^+_2}{\partial \alpha_2} - B_2 \nu^+_1 - B_1 \nu^+_2 + \theta^+_1 \theta^+_2,$$

$$E_{i3}^- = \beta_i - \theta_i,$$

$$\beta_i = \frac{1}{h} (\nu^-_i - \nu^-_i), \quad \theta^+_i = k_i \nu^-_1 - \frac{1}{A_1} \frac{\partial \nu^-_3}{\partial \alpha_i},$$

$$\theta^-_i = \frac{1}{2} (\theta^+_i + \theta_3^-).$$

Multiplying the first two of the equilibrium Eqs. (1) by the functions of the shape $$N^{+}(\alpha_3)$$ and integrating the resulting equations along with the third equation with respect to the transverse coordinate from $$\delta^-$$ to $$\delta^+$$ with allowance for the boundary conditions at the face surfaces $$S^+$$, we obtain the expressions consistent with the virtual work principle. Here, $$p^\pm_\alpha$$ are the components of the surface-load vectors $$p^\pm$$ at the face surfaces $$S^\pm$$ (see figure). As a result, taking into account Eqs. (3) and (4), we arrive at the following nonlinear equilibrium equations for the shell with respect to the stress and moment resultants:

$$\frac{1}{A_i} \frac{\partial H^+_{ij}}{\partial \alpha_i} + \frac{1}{A_j} \frac{\partial H^+_{ij}}{\partial \alpha_j} + B_i (H^+_{ij} - H^+_{ji}) + 2B_j H^+_{ij}$$

$$+ k_i S^+_{i3} + \frac{1}{h} T_{i3} = \mp p^+_i (i \neq j),$$

$$\frac{1}{A_1} \frac{\partial N^+_{i3}}{\partial \alpha_1} + \frac{1}{A_2} \frac{\partial N^+_{i3}}{\partial \alpha_2} + B_1 N^+_{i3} + B_2 N^+_{i3}$$

$$= -k_1 T_{11} + k_2 T_{22} = p_3^- - p^+_3,$$

where $$T_{ia}, \ H^+_{ij}, \ H^+_{ij}, \ N^+_{i3},$$ and $$S^+_{i3}$$ are the stress and generalized moment resultants determined by the formulas

$$N_{i3} = T_{i3} - \theta_1 H^+_{i1} - \theta^+_1 H^+_{j1} - \theta^-_2 H^+_{i2} - \theta^+_2 H^+_{j2},$$

$$S^+_{i3} = \frac{1}{2} T_{i3} - \theta_1 H^+_{i1} - \theta^+_1 H^+_{j1} - \theta^-_2 H^+_{i2} - \theta^+_2 H^+_{j2},$$

$$S^-_{i3} = \frac{1}{2} T_{i3} - \theta_1 H^+_{i1} - \theta^+_1 H^+_{j1} - \theta^-_2 H^+_{i2} - \theta^+_2 H^+_{j2},$$

$$T_{ia} = \int \sigma_{ia} d\alpha_3, \quad H^+_{ij} = \int \sigma_{ij} N^+_{i3}(\alpha_3) d\alpha_3,$$

$$H^+_{ij} = \int \sigma_{ij} [N^+(\alpha_3)]^{p+q} [N^+(\alpha_3)]^{p+q} d\alpha_3,$$

$$p, q = 0, 1.$$

The elasticity relationships for the stress and generalized moment resultants (7) with allowance for Eqs. (2) and (4) can be represented in the form

$$H^+_{0j} = \frac{1}{12} h \sum_{l \leq m} b_{ijlm} (3 E^+_{lm} + E^-_{lm}),$$

$$H^+_{0j} = \frac{1}{12} h \sum_{l \leq m} b_{ijlm} (E^-_{lm} + E^+_{lm}),$$

$$H^+_{ij} = \frac{1}{12} h \sum_{l \leq m} b_{ijlm} (E^-_{lm} + E^+_{lm}),$$

$$H^+_{ij} = H^+_{ij} + H^+_{ij}, \quad H^+_{ij} = H^+_{ij} + H^+_{ij},$$

$$T_{ij} = H^+_{ij} + H^-_{ij}, \quad \alpha_{i3} = h \sum_{i} k_i b_{i3lm} e_{i3},$$

where we take $$k_{ij} = 1$$ for the shear correlation factors. Formula (8) for the transverse forces $$T_{ij}$$ has simple meaning: the elasticity relationships for transverse shear stresses (2) in the theory of Timoshenko shells are.